1. A&M 22–1
2. A&M 22–2
3. A&M 22–3
4. A&M 22–5, parts a, b, and c only. Hint for part a: Use the chain rule to show
   \[ \frac{\partial \phi}{\partial R_\mu} = \phi' \frac{\partial R}{\partial R_\mu} = \phi' \frac{R_\mu}{R} \text{ etc.} \]
   where \( R^2 = \sum \mu R_\mu^2 \).

5. Essentially Kittel (7th) 4–7: Consider a simple model of soft phonon modes: Consider a line of ions of equal mass but alternating in charge, with \( q_m = (-1)^m e \) as the charge on the \( m \)th ion. Their interatomic potential is the sum of two contributions: 1) a short-range interaction of force constant \( K_1 R = \gamma \) that acts between nearest neighbors only, and 2) a Coulomb interaction between all ions.

a) Show that the contribution of the Coulomb interaction to the atomic force constants is \( K_{mc} = 2 (-1)^m e^2 / m^3 \alpha^3 \), where \( \alpha \) is the equilibrium nearest-neighbor distances.

b) Using the general 1–D dispersion relation \( \omega^2 = (2/M) \sum_{m=1}^{\infty} K_m (1 - \cos mk) \) [eqn. (22.90)], show that the dispersion relation for this specific system can be written as
   \[ \frac{\omega^2}{\omega_0^2} = \sin^2 (ka / 2) + \sigma \sum_{m=1}^{\infty} (-1)^m [1 - \cos(mka)] / m^3, \]
   where \( \omega_0^2 = 4\gamma / M \) and \( \sigma = e^2 / \gamma a^3 \).

c) Show that \( \omega^2 \) is negative (i.e. the mode is unstable, or "soft") at the zone boundary
   \( ka = \pi \) if \( \sigma > 0.475 \) [i.e. \( 4 / [7\zeta(3)] \)], where \( \zeta \) is the Riemann zeta function.

d) Show that the speed of sound at small \( ka \) is imaginary if \( \sigma > 1 / (2 \ln 2) \approx 0.721 \).

6. Using http://dept.kent.edu/projects/ksuviz/leeviz/phonon/phonon.html (or analytically), consider a diatomic chain. Take \( ka = 0.5 \) and set the mass ratio to 4.

a) Find the ratio of the amplitudes of the vibrations of the two atoms in i) the optical and ii) the acoustic branch.

b) What is the ratio of the period of the acoustic mode to that of the optical mode? For what value of \( ka \) is this ratio about 2 (with mass ratio fixed at 4)?