Homework #3 — Phys625 — Spring 2004
Deadline: Wednesday, April 28, 2004.
Turn in homework in the class or put it in the box on the door of Phys 2314 by 3 p.m.

Victor Yakovenko, Associate Professor
Office: Physics 2314
Phone: (301)-405-6151
E-mail: yakovenk@physics.umd.edu

Web page: http://www2.physics.umd.edu/~yakovenk/teaching/phys625.spring2004

Do not forget to write your name and the homework number!
Equation numbers with the periods, like (3.2.25), refer to the equations of the textbook.
Equation numbers without period, like (5), refer to the equations of this homework.

Density-density correlation function

1. *Plasmon excitation*

(a) [**2 points**] Using the identity $f^+_{\mathbf{p}+\mathbf{q}} = 1 - f^-_{\mathbf{p}+\mathbf{q}}$ and the symmetry with respect to interchange of $\mathbf{p}$ and $\mathbf{p}+\mathbf{q}$, show that the real part of Eq. (6.5.11) can be rewritten as

$$\mathrm{Re}\,\mathcal{P}_0(\mathbf{q},\omega) = 2\int \frac{d^3p}{(2\pi)^3} f^-_{\mathbf{p}} \frac{2\omega_{\mathbf{q}}}{\omega^2 - \omega^2_{\mathbf{q}}} \tag{1}$$

(b) [**2 points**] From Eq. (1), obtain an asymptotic expression for $\mathrm{Re}\,\mathcal{P}_0(\mathbf{q},\omega)$ in the limit of small q, but finite ω.

(c) [**2 points**] Substituting the expression for $\mathrm{Re}\,\mathcal{P}_0(\mathbf{q},\omega)$ found in Problem 1b into Eq. (6.5.17), determine the frequency ω_p where the dielectric susceptibility $\epsilon(\mathbf{q},\omega)$ has a pole at $q \to 0$. This is the frequency of the plasmon excitation.

2. *Classical plasma frequency* [**2 points**]

Let us derive the frequency of plasma oscillations in a different (classical) way.

Consider the continuity equation for the electron gas:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) \approx \frac{\partial\,\delta n}{\partial t} + n_0\nabla\cdot\mathbf{v} = 0, \tag{2}$$

where $n(\mathbf{r},t) = n_0 + \delta n(\mathbf{r},t)$ is the concentration of electrons, separated into the equilibrium uniform concentration n_0 and the deviation δn, and $\mathbf{v}(\mathbf{r},t)$ is the velocity of hydrodynamic flow.

Let us differentiate Eq. (2) in time:

$$\frac{\partial^2\delta n}{\partial t^2} + n_0\nabla\cdot\frac{\partial\mathbf{v}}{\partial t} = 0. \tag{3}$$

Using Newton's equation of motion $\partial\mathbf{v}/\partial t = -e\mathcal{E}/m$ and Poisson's equation $\nabla\cdot\mathcal{E} = -4\pi e\,\delta n$, where $\mathcal{E}$ is electric field, transform Eq. (3) to the following form

$$\frac{\partial^2\delta n}{\partial t^2} = -\omega_p^2\,\delta n \tag{4}$$

and determine the frequency ω_p. Eq. (4) describes local oscillations of electron density. Compare the plasma frequency ω_p in Eq. (4) with the expression obtained in Problem 1c.

3. *1D density-density correlation function*

(a) [**4 points**] In the 1D case, take the integral in Eq. (6.5.11) or in the unnumbered equation before it and explicitly calculate $\mathcal{P}_0(q,\omega)$.

(b) [**2 points**] Using the expression obtained in Problem 3a, sketch a plot of the static $(\omega = 0)$ density-density correction function $\mathcal{P}_0(q,0)$ vs. q. Discuss its behavior at $q \to 2k_F$.

(c) [**4 points**] Using the expression for $\mathcal{P}_0(q,\omega)$ obtained in Problem 3a and the formula

$$\ln(x) = \ln|x| + i\pi\theta(-x), \tag{5}$$

determine the area in the $(q > 0, \omega > 0)$ plane where $\mathcal{P}_0(q,\omega)$ has a nonzero imaginary part. Compare your result with the solution of Problem 3 in Homework 2.

(d) [**4 points**] From the expression obtained in Problem 3a, find an asymptotic expression for $\mathcal{P}_0(q,\omega)$ the limit of small $q \ll k_F$ and small $\omega \ll E_F$. Looking at the poles in the found expression, describe the spectrum of excitations in this limit.