Theories and Practices of Thinking and Learning to Think

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One of the persisting aims in mathematics education is that students become more able in mathematical thinking. This article discusses relations between research about processes of learning and thinking, and educational practices that attempt to achieve that aim. We discuss three research perspectives that characterize thinking and learning to think differently. The *associationist/behaviorist* perspective views learning to think as acquisition of higher-order skills. The *domain-structural/cognitive* perspective views learning to think as acquiring schemata and strategies for understanding and reasoning, with concepts in subject-matter domains and with general principles of reasoning and problem solving. The *situative* perspective views learning to think as becoming a more effective participant in social practices of inquiry and sense-making, in which individuals develop their identities as learners and knowers. These views of thinking are reflected in educational practices that emphasize acquisition of skills, conceptual understanding and thinking strategies, and participation in practices and development of identity. The Middle-School Mathematics through Applications Project is an example of learning environments, curricula, and teaching that are designed with the situative focus. We argue that the topic of thinking provides an example in which the situative perspective can provide a framework that includes the strengths and values of the behaviorist and cognitive perspectives.

In general, we believe that research influences educational practice mainly by informing discourses of practice. Researchers participate in discussions of the curriculum and in discussions with teachers and other practitioners. Findings and concepts that are developed in research are discussed within communities of educational practitioners, especially teachers, whose understandings of thinking and learning shape their decisions of what and how to teach and their interactions with students.

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Research also influences discourse about education in the larger society, in which expectations for educational practice are developed and sometimes translated into policies that affect resources for and constraints on practitioners' activities.

One way that research informs discourses of practice involves general conceptions in the domain of activity in which the practice functions. Researchers develop more refined and coherent versions of concepts, holding each other accountable for developing representations of concepts that can be used consistently and coherently to explain phenomena in their domain of study. Different conceptions of what it means to know, think, and learn correspond to different assumptions about what education does and what it should accomplish (cf. A. L. Brown 1994; J. S. Brown 1991). When people—researchers as well as others—discuss the aim of students' learning to be better mathematical thinkers, the meanings of these concepts matter, and considerations of consistency and coherence of explanations come into the discourse. To a significant extent, the assumptions that underlie educational practices are taken from everyday conceptions. For example, the dominant image of thinking in our society is conveyed by Rodin's *The Thinker,* which depicts thinking as an isolated, serious, and perhaps even painful activity.

A second way that research informs discourses of practice is through demonstrations and analyses of possibilities. Demonstrations of learning with nonstandard materials and methods can contribute to practitioners' understandings of the spaces of alternatives in which they design and choose alternative practices and programs. Analyses of these examples can help practitioners understand why the examples worked and failed to work as they did and provide a basis for deciding what they need to do in their own circumstances to achieve their own aims.

We organize our discussion with three general perspectives regarding

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the nature of knowing, thinking, and learning, that have been important in American thought and educational practice. These are the *associationist/behaviorist* perspective, the *domain-structural/cognitive* perspective, and the *situative* perspective (cf. Case 1992; Greeno et al. 1996; Greeno and the Middle-School Mathematics through Applications Project Group, in press). Each of these perspectives provides a view of learning and thinking that underlies educational practices in which students can engage in thinking and learning to think. The perspectives bring different aspects of learning and thinking into the foreground. The associationist/behaviorist perspective emphasizes the development of skill; the domain-structural/cognitive perspective emphasizes conceptual understanding and strategies of problem solving and reasoning; and the situative perspective emphasizes participation in practices of inquiry and sense-making of a community, and development of individual's identities as thinkers and learners.

All of these perspectives can be used in understanding educational practices and processes of many kinds. We do not believe that any particular view of learning and thinking implies that some practices are correct and others are not. That is, we do not claim that there are associationist practices, cognitive practices, and situative practices as such. The relation between theory and practice is more subtle than that. As we understand that relation, it depends on differences between the perspectives in what they emphasize about learning and thinking. Any educational practice can be considered from any of these points of view. Different practices are encouraged by research in the different perspectives, however, because practices are organized to emphasize different educational aims. The perspectives bring different aspects of practice into focus and those different foci correspond to significantly different emphases in practices.

The next sections of our article discuss the three theoretical perspectives in turn. In each section, we discuss assumptions about thinking and educational practices that focus on the aspects of thinking, and learning to think, that are emphasized in that perspective. These three perspectives have been represented in the discussions of education for many years, and we quote an American representative of each perspective from the first half of this century. In the section on the situative perspective, we discuss our own curriculum development and research project, the Middle-School Mathematics through Applications Project (MMAP), as an example of research in the situative perspective based on curriculum materials that are designed with the situative focus on participation in practices and development of students' identities as mathematics thinkers and learners. We argue, in our conclusion, that the situative perspective is inclusive of the others. That is, by viewing learning and thinking as aspects of participation in social practice and individual identity, we can include the significant strengths and values of learning skills, conceptual understanding, and thinking strategies.

Skills for Thinking: The Associationist/Behaviorist Perspective

Conceptualizations of Learning and Thinking

In the psychological theories of associationism and behaviorism, knowing is an organized accumulation of associations between ideas or between stimuli and responses. Thinking, in this view, is the activation of ideas or performance of responses that are instrumental in overcoming obstacles to the thinker's purposes and goals and that lead to reinforcing states of affairs. Abilities to think are considered as higher-order skills that depend on having more basic skills that can be brought into play in solving complicated problems. This assumption is consistent with school practices in which students are expected to accumulate basic skills, but are not expected to use those skills in thinking about complex problems, concepts, or situations. Of course, thinking is a valued ability, and it is hoped that students will eventually be able to think about complicated things, but the activities of school learning are mainly organized so that students can accumulate the skills that they need to think with, rather than presenting them with problems that present challenges for complex thinking for which they are assumed not to be ready.

Associationist discussions of thinking were presented by William James (1890) and Edward Thorndike (1905). The general idea was that thinking could be spontaneous or purposive. In spontaneous thinking, ideas "flow on at random, unchecked by any interference on the part of our general intentions and aiming at no desired goal," while "in controlled thinking each thought as it comes is attended to; its usefulness is judged in the light of our general system of ideas and purposes concerning the topic in hand; it is allowed to remain and influence the future course of thought only when it seems fit" (Thorndike 1905, p. 264). James (1890) emphasized analysis and abstraction in thinking, by which he meant attending selectively to aspects of a state of affairs that are pertinent to one's purposes in thinking and mentally separating those aspects from the accompanying, irrelevant aspects. Thorndike (1931) emphasized, in addition, that thinking requires putting things together "in the proper relations." In Thorndike's view, the main impediments to successful thinking are irrelevant associations.

> A person who considers an intellectual problem and gets an answer or solution for it is beset by tendencies of literally every element in the problem to assume undue potency, to become dislocated from its proper relations, and to call up its own associations with too little regard for the general mental set or adjustment. If the person does get the right solution, that means that an elaborate hierarchy of connections is in operation and that a very delicate balance of power is maintained. Most, if not all, of the tendencies which produced the errors of our illustrations existed also in the pupils who made no such errors. The tendencies were there, but were prevented from determining final response by other competing and cooperating tendencies.
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> The compositions of forces which determine the direction of thought are thus highly elaborate and complex; but the forces themselves are very simple, being the elements in the situation and the connections leading from those elements and various combinations thereof which the past experience and present adjustment of the thinker provide.
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> The simple facts of over-potency and under-potency, dislocation of elements into wrong relations, and imperfect or erroneous associations are all that are needed to explain errors in thinking. Conversely, the right weighting of elements, held in the right relations, and connected with the right associates, explains correct thinking. (Thorndike 1931, pp. 159–60)

The primary concern of behaviorist theorists, such as John Watson (1914) and B. F. Skinner (1957), was to argue that behaviorist concepts are broad enough to include explanations of thinking. They conceptualized thinking as covert behavior, including, but not limited to, covert speech. Skinner's discussion focused on the mechanisms of reinforcement that could plausibly be assumed to maintain thinking behavior. He hypothesized that in conversations, listeners provide reinforcement for speakers as they indicate understanding and agreement, and that in the covert verbal behavior of thinking, functions of the speaker and listener are combined in the thinking individual, with the effect that the behavior is self-reinforcing.

Behaviorist theorists who worked in Clark Hull's (1930) program gave more systematic attention to questions about thinking. Irving Maltzman (1955) and others engaged in a productive debate in which they responded to challenges by gestalt psychologists such as Karl Duncker (1945), George Katona (1940), Wolfgang Köhler (1929), and Max Wertheimer (1959). Much attention was given to Duncker's functional fixity experiments in which he showed that the solution of a construction problem could be made much less likely if a critical object was being used or had been used just previously in another way. For example, in a problem to be solved by using a box to support a candle, the problem is much harder if the box is already being used to hold tacks than if it is just sitting on the table.

Behaviorists interpreted these phenomena in terms of stimulus-response associations, of course. Their accounts of thinking included hypotheses about covert responses and their internal stimulus consequences, an idea that was used generally in Hullian behaviorist analyses. Explanations of problem solving also hypothesized that tendencies to respond are organized in levels, with more general tendencies as well as tendencies that depend on specific stimulus conditions (Maltzman 1955). Phenomena of functional fixity were explained by hypothesizing that a covert response of relating the needed object to its potential use in the problem is less likely to occur because of competition from a stronger response of relating it to its present function. (James could have said that the properties that enable its needed function are not abstracted, and this could be interpreted by Thorndike to be caused by attending to the object's current function with undue potency, with too little regard for the "general system of ideas and purposes concerning the topic in hand.") Behaviorists supported their analyses with experiments that supported predictions based on hypotheses about covert competing responses, and experimental variables that were known to affect response strength, such as time since a response had been reinforced and degree of motivation (e.g., Adamson and Taylor 1954; Glucksberg 1962).

An interesting contrast between latter-day behaviorism and early connectionism involved emphasis given to what Thorndike called controlled and spontaneous thinking. The phenomena demonstrated by gestalt psychologists, including functional fixity and mental set (Luchins 1942), focused attention on difficulties of solving problems in which a successful response is available, but the problem solver is unlikely to perform it. Both gestalt and behaviorist psychologists concluded that efforts to improve thinking should be addressed to people's tendencies to respond too narrowly. Abraham Luchins's experiment and discussions made the point very clearly. He gave a series of problems that were solved by the same sequence of arithmetic operations. When he then gave a problem that could be solved either with that sequence of operations or a much simpler one, participants persisted with the sequence that they had used previously. Luchins referred to this as mechanization of problem solving, and considered it a deficiency of human cognition. Much attention was given to ways of teaching people to think flexibly, rather than being limited to familiar associations and patterns. For example, Maltzman (1960) developed a method of reinforcing participants for giving unusual responses in a word association test, and showed that this increased their likelihood of solving some problems.

Educational Practices Aligned with the Associationist/Behaviorist Focus

The associationist/behaviorist conception of learning has had profound influences on educational practice in the United States. In the years following World War II, much educational research and development emphasized ideas and technologies that were primarily behavioristic. The idea that thinking correctly depends on having learned responses at multiple levels that support using appropriate general approaches in solving problems and understanding was especially clear in Robert Gagné's (1965) discussions of instructional analysis and design. The theory and technology of task analysis had been developed and used successfully in personnel selection and training. Much educational practice became committed to organizing curricula and assessments according to behavioral objectives, based on analyses of complex skills into their components, and with instruction organized to ensure that students could master the prerequisites of material they were expected to learn (Gagné 1968). There was considerable optimism that a well-ordered system of curriculum and instruction, based on scientific analysis, could make basic knowledge available to virtually all of America's children (Bloom 1976). In this view of learning, differences between students in their rates of progress through the curriculum are considered as difficulties to be overcome, and technologies that can be used to present material and tasks in sequences tailored to individuals' progress have been emphasized (Skinner 1958; Suppes and Morningstar 1972). This scientific and practical optimism encouraged the broadening of the nation's commitment to providing equitable educational opportunities to its children in the Great Society programs of the 1960s and 1970s.

It would be hard to miss the fact that most students do not learn to think as successfully as we would like, especially in domains such as mathematics and science. Historically, behaviorists and associationists have had two responses to this fact, one based on an assumption about individual talent and/or motivation, and the other based on a concept of thinking as a general skill that might be learned.

According to the assumption about individual talent and/or motivation, the cause of most students' lack of success in thinking, say, in mathematics, is their lack of mathematical talent or motivation to expend the effort needed to think about complicated things. This is a widespread view in the society; an especially clear version of it was articulated by Henri Poincaré (1956). It is assumed that only a few students have the necessary talent or motivation for complex thinking, and that as the material to be learned becomes more complex, the deficiencies of the less talented students become evident. One implication of this view is that if mathematical talent can be measured in students before they have learned the basic skills, then it can be productive to provide thinking challenges to students who have been identified as being gifted and talented. The opportunity to participate in such classes may provide increased motivation for gifted students who might otherwise not develop an interest in the subject, but students without the necessary talent are assumed not to be able to profit from such challenges.

Alternatively, according to the assumption that thinking depends on a general skill, as well as on accumulated basic knowledge, the cause of most students' success in thinking is their lack of acquiring this skill. One possibility is that people need to develop a kind of second-order habit to perform unusual responses, and methods were developed for training people to give associations to words that were not the most common ones (Maltzman 1960). Paul Torrance (1966) developed tests for creativity measured as the person's ability to think of unusual uses for objects, and training programs were developed for business executives in which they learned to brainstorm (Osborn 1953) or think analogically (Gordon 1961), rather than being limited to ideas that are obviously relevant to the problem. In a program designed for school use, Martin Covington et al. (1974) developed activities in which students are encouraged to try out their ideas, and to develop techniques for discovering and formulating problems and restructuring their approaches to problems, rather than limiting their actions to those that they have learned to use to solve specific kinds of problems.

Structures of Thinking: The Domain-Structure/Cognitive View

Conceptualizations of Learning and Thinking

In theories that emphasize concepts and reasoning methods in subject matter domains and cognitive processes, knowing is a structure of mental representations, which is organized according to general principles of reasoning and to concepts and principles of a domain of knowledge. In this view, correct and successful thinking depends on (*a*) whether the individual's cognitive structures are consistent with correct concepts, according to general principles of logic, mathematics, and accepted knowledge in subject matter domains, and (*b*) whether the individual has acquired appropriate procedures and strategies for applying her or his knowledge structures in solving problems and reasoning.

The domain-structural/cognitive view was represented in educational psychology by Charles Judd, for example, in his discussion of an experiment in which teaching boys some aspects of the concept of refraction resulted in their having better transfer from learning to aim at a target under water to a new situation with a lower level of water, than did a group that only practiced the skill in initial learning (Judd 1908). Regarding educational practice, Judd argued against both a pedagogy of memorization and a pedagogy organized entirely by activities of ordinary experience. He opposed "the extreme position that subject-matter courses should be abandoned altogether in favor of projects which engage a child's activities and arouse his thinking without adhering to conventional lines," as well as the proposal "that the curriculum be reconstructed so as to provide much handwork, that pictures be substituted for reading material, or that excursions take the place of ordinary recitations" (Judd 1936, p. 168). He based his advocacy of including "systematically organized subject-matter courses" on an analysis of thinking.

> The effort of educational psychology to reach a decision with regard to the retention or rejection of systematic courses must begin by considering what happens when an individual is confronted by a strange experience. The first impression which an individual receives from an unfamiliar landscape, for example, is a vague, somewhat confused view including in a loose aggregation a large number of items. The first stage of experience is followed by a period of analysis. The eye rests on one item in the landscape after another, and the observer cultivates discriminating familiarity with the details of various objects. After analysis has made the mind familiar with certain items, a process of organizing synthesis begins. In this third stage of experience the observer sees in their proper relations the objects that make up the total scene. Each object takes its place as a part of the whole and yet retains its individuality.
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> It is the third, or last, stage which is to be described as the stage of true comprehension. The first stage is characterized by breadth and inclusiveness but not by organization; it lacks orderly arrangement. . . .
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> Education offers to the maturing individual not only masses of experience but also complex devices, such as language and number, which aid in organizing these masses of experience. . . .
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> A familiar example appears in the instruction which the high school gives—or aims to give—in geometry. This science prepares the pupil to think about space relations. Some of the facts about space are known to the pupil through the long, exploratory contacts

> which he has had with distances, sizes, and shapes in the course of his ordinary experience. Geometry takes the spatial figures known to the pupil and makes a detailed and orderly analysis of these figures. The triangle is treated with reference to its angles and sides. After the triangle has been thoroughly mastered by the pupil, other figures are analyzed. In order to facilitate concentration of attention on the spatial characteristics of figures, geometry neglects the fact that figures are made of lines on the blackboard or on paper or that they are the outlines of objects made of wood or stone. The mind is acquiring a method of analysis and synthesis in a single realm, namely, the realm of spatial form. It must be free from the distractions which would arise if it attempted at the same time to perform various other types of analysis, such as analysis of materials. (Judd 1936, pp. 168–71)

Piaget (1970) did not share Judd's enthusiasm for school-taught formal methods in thinking, but he did share Judd's emphasis on the need for cognitive resources that organize experience systematically. The operational structures that Piaget hypothesized, such as conservation and proportional reasoning, provide bases for relating aspects of situations and making inferences.

Subsequent to Piaget, research on cognitive development has focused on the growth of conceptual structures in domains such as number (Gelman and Gallistel 1978), physical objects and interactions (diSessa 1993; Spelke 1990), life (Carey 1985; Hatano and Inagaki 1996; Keil 1989), and the mind (Flavell et al. 1986; Wellman 1990). This research has identified conceptual capabilities in children's intuitive understanding of events and systems that they experience and suggests that educational research should focus on these constructive capabilities as providing a basis for students' coming to understand the principles of standard scientific and mathematical accounts (Smith et al. 1993–94). This has also been an emphasis of constructivist research and curriculum development programs (Confrey 1990; Steffe et al. 1988).

Processes of thinking have been studied in detail in the symbolic cognitive perspective, including development of detailed computer-program simulations of cognitive representations and procedures (Newell and Simon 1972). These studies have made significant headway toward representing detailed structures of and operations on information that is involved in understanding and solving problems of the kind often used in instruction in subject matter domains (Smith 1991). A significant research question involves different forms of representing information in the reasoning process, particularly whether information is represented in a form corresponding to propositions and rules (Rips 1994) or a form corresponding to models and simulations (Johnson-Laird 1983). Computational models of thinking include simulations of information and processes involved in solving specific classes of problems, as well as representations of general schemata that organize information according to domain concepts (Rumelhart and Norman 1989) and general strategies for working on recognizable kinds of problems (Greeno and Simon 1989). Models of symbolic information processing have carried forward an aspect of the main research problem addressed in gestalt psychology by characterizing organized structures of information. At the same time, as Michael Wertheimer (1985) pointed out, gestalt psychologists were primarily interested in the kind of generative insight in which someone achieves deep structural understanding without already having a procedure for representing the information as an instance of a pattern, and it is unclear, at least to us, whether information-processing systems that have been developed have made significant progress on that.

Educational Practices Aligned with the Domain-Structural/Cognitive Focus

From the 1950s into the 1980s, there were three significant developments in curricula and research about thinking consistent with the domain-structural/cognitive perspective. First, the Sputnik-fueled development of new curricula in science and mathematics organized materials and instruction around basic concepts and principles of subject matter disciplines, rather than elementary components of skill. Concerns about providing students with coherent conceptual systems were paramount in curricula developed in the 1950s and 1960s, especially in mathematics and science, for example, the reforms of the School Mathematics Study Group (1965) and the Physical Sciences Study Curriculum (Rutherford et al. 1970). In these programs, curricula were fundamentally reorganized to emphasize understanding of unifying concepts and principles of subject matter domains, rather than focusing primarily on students' learning procedures and formulas. Many advocates of this curricular change argued that stronger understanding would result in better performance on procedures.

Second, as students' learning with these more conceptual materials was observed and studied, using interview methods reminiscent of Piagetian research in cognitive development, educators and researchers found that students do not begin their studies as blank slates. Extensive study has been made of their intuitive understandings, and consistency with the conceptual structures of subject matter domains has been taken as the standard of mature reasoning, much as consistency with Piaget's operational structures has been taken as the standard of cognitive growth. Researchers have identified ways in which many students systematically deviate from the theoretical principles of science and mathematics when they make inferences about events that are described. In some research, these naive understandings have been labeled as misconceptions and fallacies (McClosky 1983; Tversky and Kahnemann 1983). Others, such as Smith et al. (1993–94), have explained these as efforts to explain phenomena using causal and quantitative patterns that are valid in many situations, but that produce invalid explanations when they are misapplied. According to this view, students' intuitive understandings can be used as resources for helping them build their understanding, rather than as impediments that need to be overcome. Numerous curriculum programs have been developed based on analyses of students' initial conceptual understandings in subject matter domains, such as density (Smith et al. 1992), heat and temperature (Wiser and Kipman 1988), mechanics (White 1993), and photosynthesis (Roth 1986).

Third, concerns with students' abilities to solve problems that go beyond the specific procedures they have been taught led to emphasis on processes and strategies of problem solving, informed by detailed studies of processes of problem solving and reasoning in cognitive science (National Council of Teachers of Mathematics 1980). Heuristic methods of solving problems of specific kinds in subject matter domains were developed and shown to be teachable (Schoenfeld 1985). Strategic aspects of problem solving have been modeled explicitly in cognitive analyses, including processes of setting subgoals and evaluating progress toward solutions. These models have been the basis of providing representations for students to recognize these strategic features and/or providing feedback and hints to students that facilitate their learning to solve text problems (Anderson et al. 1990). Curriculum developers, especially in mathematics, have given greater emphasis to applications of problem-solving methods in a variety of contexts, describing situations that students can place themselves in and using those placements to build up mathematical concepts (Usiskin 1991).

The domain-structural/cognitive perspective responds to difficulties that many students have in learning to understand and think with more specific measures than the general concerns with talent, motivation, and general thinking skills of the associationist/behaviorist perspective. Strategic aspects of problem solving can be topics of class discussion (Carpenter et al. 1989) and can be represented explicitly in intelligent computer-based learning systems (Anderson et al. 1990). Domain concepts can be related to understandings that students have initially in class discussions (diSessa and Minstrell, in press) and in computer-based learning systems (White 1993). And students can be encouraged to use learning strategies involving generating explanations that are beneficial for their learning with understanding (Chi et al. 1994).

Practices of Thinking: The Situative Perspective

Conceptualizations of Learning and Thinking

In the situative perspective, knowing is considered as sustained participation in communities of practice in which individuals develop their identities both in relation to their contributions to the community's functions and progress and in relation to their activities and growth as individuals. Thinking is an aspect of social practice, involving reflection and discourse on activities of individuals and groups and of meanings of concepts that are significant in evaluating and making sense of the community's and of individuals' activities and experiences. The situative perspective considers processes of thinking as participation in practices of communities and the development of individuals' identities as learners and thinkers. In this view, learning to think is increasing one's ability to participate effectively in the practices of a community that involve inquiry and reflective analysis. The key idea is that students learn to think by participating in activities where they actively engage in thinking. This idea was articulated by Dewey, who argued that if the main goal of education is development of students' abilities to think, then students must learn in situations where thinking has a chance to occur.

> What is here insisted upon is the necessity of an actual empirical situation as the initiating phase of thought. Experience is here taken as previously defined: trying to do something and having the thing perceptibly do something to one in return. The fallacy consists in supposing that we can begin with ready-made subject matter of arithmetic, or geography, or whatever, irrespective of some direct personal experience of a situation. . . .
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> Hence the first approach to any subject in school, if thought is to be aroused and not words acquired, should be as unscholastic as possible. . . . Careful inspection of methods which are permanently successful in formal education, whether in arithmetic or learning to read, or studying geography, or learning physics or foreign language, will reveal that they depend for their efficiency upon the fact that they go back to the type of the situation which causes reflection

> out of school in ordinary life. They give the pupils something to do, not something to learn; and the doing is of such a nature as to demand thinking, or the intentional noting of connections; learning naturally results. (Dewey [1916] 1966, pp. 153–54)

For Dewey, the main goal of education was for students to acquire habits of mind, which in the terms of our discussion here are the means of participating in practices of inquiry.

> Processes of instruction are unified in the degree in which they center in the production of good habits of thinking. While we may speak, without error, of the method of thought, the important thing is that thinking is the method of an educative experience. The essentials of method are therefore identical with the essentials of reflection. They are first that the pupil have a genuine situation of experience—that there be a continuous activity in which he is interested for its own sake; secondly, that a genuine problem develop within this situation as a stimulus to thought; third, that he possess the information and make the observations needed to deal with it; fourth, that suggested solutions occur to him which he shall be responsible for developing in an orderly way; fifth, that he have opportunity and occasion to test his ideas by application, to make their meaning clear and to discover for himself their validity. (Dewey [1916] 1966, p. 163)

The sources of learning to think and understand by participating in social interaction were emphasized by L. S. Vygotsky (1987) and George Herbert Mead (1934). Recent research that has developed concepts of thinking as collaborative social practice includes studies by Elinor Ochs et al. (1994) and by Lucy Suchman and Randy Trigg (1993), of groups of scientists interactively constructing and interpreting diagrams as they discuss the design and interpretation of experiments or computer programs. Similarly, recent research in the philosophy of mathematics and science and in science studies (Kitcher 1983, 1993; Latour and Woolgar 1986; Longino 1990; Pickering 1992; Tymoczko 1985) have begun to develop understandings of knowledge in the sciences as products of practices that attend especially to criteria for acceptable inferences.

Many studies of everyday reasoning have provided evidence and analyses that show ways in which the concepts and methods of mathematics function in important ways. Usually, mathematical concepts are implicit in everyday reasoning, rather than being represented explicitly (Hutchins 1995; Lave 1988; Nunes et al. 1993; Saxe 1990).

Educational Practices Aligned with the Situative Focus

Educational studies that take this situative perspective often create environments in which students can learn to participate in practices of productive inquiry and use of concepts and principles that are characteristic of subject matter disciplines.

An early example was Harold Fawcett's (1938) course in geometry, in which students participated in formulating definitions, choosing postulates, and constructing and evaluating arguments as their main classroom activity. Fawcett and his students gave considerable attention to the concepts of geometry—each student constructed a version of geometry that developed definitions, postulates, and theorems. They also attended to applications of methods of proof to everyday examples. Their activity was focused much more on the practices of constructing and evaluating arguments than are those of the concept-oriented or skill-oriented programs of the domain-structure/cognitive or associationist/behaviorist perspectives.

Recent studies that take this situative perspective include development and study of classroom activities that are organized for students to participate in the discourse of the subject matter, including formulating and evaluating hypotheses, conjectures, arguments, evidence, examples, and conclusions (Cobb et al. 1993; Cohen et al. 1993; Hatano and Inagaki 1991; Lampert 1990; Schoenfeld 1994). They also include projects that develop materials, classroom activity structures, and teaching practices in which students work on extended projects as communities of learners (Brown and Campione 1994; Cognition and Technology Group at Vanderbilt 1994; Scardamalia et al. 1994), including the MMAP program at the Institute for Research on Learning and Stanford, which we discuss in the next section.

In the situative perspective, difficulties that students have in learning to think and understand are interpreted as impediments to their participation in social practices. Participation is a relation of the individual and the community and its activities, and engaged participation can be facilitated in numerous ways. Many current studies are finding that students' engagement and effectiveness are enhanced when the participation structures of classrooms are changed to encourage their more active participation in practices of inquiry and sense-making. In more open-ended participation structures and with activities designed to be accessible to students on the basis of their experience outside of school, diversity of students' current understanding can be an essential positive resource for learning in the classroom community. The question shifts from, "Does this student have the talent and motivation to learn these lessons?" to "What can this student contribute to the discussion and to the class's or group's progress on its project?" The latter question has a positive answer more frequently than the former question (cf. Moll and Whitmore 1993).

An Illustrative Project

The MMAP illustrates educational development and research that emphasizes students' participation in practices of learning, reasoning, and inquiry. As stated earlier, the situative perspective is a commitment to focus on participation in practices in designing and analyzing educational practices. When attention is focused on the practices that students participate in, people may decide that they prefer practices that are different from the ones that seem most effective if they focus on skills or conceptual understanding, but all learning is situated in some practices, and a preference for practices of relatively open-ended inquiry is a consequence of valuing students' learning to participate in those activities, rather than an implication of the situative view of learning and thinking.

Goals and activities.—Our main educational goal is to design and develop resources for all students' learning of mathematics. By "learning of mathematics," we mean becoming effective participants in practices of mathematical thinking and acting that include communicating, inquiry, reasoning, and understanding. We assume that arranging for all students to learn mathematics in this sense requires a major change in standard educational practices, and we take it as our job to facilitate teachers' and students' developing different teaching and learning practices when using the new materials. Our educational approach is to present mathematics primarily as a resource for real-world problem solving. More specifically, the students' leading activities are projects in which they design simulations of real things in the world, with mathematics providing concepts and other tools for aspects of their work. This contrasts with standard practice in which mathematics provides the topics of activity, with peripheral attention to various ways in which mathematical concepts and methods can be used. The activities that we have developed are intended to be accessible and interesting to middle school students, and to involve use of mathematical concepts that are appropriate for students of these ages.

Our program for advancing the educational goal of fostering students' learning to think mathematically is supported in a general way by research on thinking viewed as collaborative social activity, such as the studies we mentioned briefly above. Research on the processes of thinking, viewed as social practices, is less well developed than research on thinking viewed as individual cognitive activity, although there are promising beginnings (Greeno and Goldman, in press). By studying the activities of students and teachers using the MMAP materials, we are developing information and theoretical concepts that contribute to the scientific understanding of fundamental processes of thinking and learning to think.

Our main research goals are to analyze and understand how the classroom activities that occur with these materials are organized. We are especially concerned with ways in which mathematical understanding and reasoning occur explicitly and implicitly in the discourse among students and between students and teachers.

Projects of this kind are necessarily collaborative, requiring contributions from participants who are members of several different professional communities. Active participants in the project include several senior educational researchers, four additional technical/professional staff, about six post- and predoctoral researcher/developers, and 20–30 teachers.

Work activities in the project are equally various. They include designing, developing, evaluating, and modifying computer programs and curriculum activities that are used in classrooms; videotaping students and teachers in their use of the materials in selected classrooms and examining and analyzing materials on a subset of these videotape records; designing and developing materials about the curriculum and teaching for use by teachers, other than our participating teachers, who want to adopt MMAP materials; and videotaping, examining, and analyzing records of our activities of collaborative design and development of these materials. They also include visits that our participating teachers make to sites in which they observe professionals who use mathematics in their work—such as civil or product-design engineers, architects, environmental or space scientists, and home repair specialists—and discussions about the activities of these professionals regarding their understanding and use of mathematics.

Learning resources and activities.—The work of MMAP in developing learning materials and curricula involves developing computer programs that function as simulation and modeling environments for middle school students and composing curriculum guidelines for teachers to organize projects that students carry out with the assistance of the technology. We have developed computer-aided design (CAD) systems and curriculum sequences for four problem-solving domains.

One CAD system, called ArchiTech, supports designing floor plans of buildings. A graphics interface supports construction of floor plans with icons for walls, doors, windows, stairways, and furniture. The program calculates several quantitative properties of a design, including area of the floor space, quantities and costs of building units, such as linear meters of walls and numbers of windows and doors, and estimated total building and heating costs based on the quality of insulation and assumptions about indoor and outdoor temperature that the students set. Data for several different design iterations are entered in a data chart that resembles a spreadsheet. An extensive curriculum sequence for ArchiTech, called the Antarctica Project, develops a project of designing a living and working space for a small group of scientific researchers who will work in Antarctica for two years. Another curriculum sequence for ArchiTech is called Dream House, which is what students design. In both programs, students work on their design projects in small groups and prepare reports of their designs with analyses of cost factors and other supporting material. We have also constructed a variety of assessment tasks for each curriculum program. In one assessment activity, students are asked to estimate the size of a table in their room and to draw this table on the computer screen using two different scales.

A second environment, called HabiTech, supports designing quantitative models of ecological systems. Students construct models that simulate growth and decline of species by specifying initial quantities such as population sizes, and causal parameters, such as birth rates, death rates, and their timing, due to natural causes and predation. The graphics interface supports construction of simple systems-theoretic networks that represent the population quantities and their interactions. The computer program constructs tables and graphs to represent changes in population sizes over time, based on assumptions that the students have included in their models. One curriculum program for HabiTech, called Lifelines, describes a policy issue of whether to have wolf hunting in controlled areas of land in Alaska. The students' project is to develop a model or set of models based on research data on wolf populations that will inform the policy decision. Their activities include exploring implications of alternative assumptions about causal interactions and rates and preparing and giving reports that present their findings and conclusions.

The third software environment that we have developed is Coding Toolbox, which is a program for encryption that allows students to create codes by specifying functions from letters to numbers, using ciphers, matrices, and compression patterns. Students take on the roles of code makers and code testers for "clients" who have specific needs for codes. They might have a client who needs a secret code or a client who needs to compress messages that will be transmitted over a network. Students' activities include creating codes to meet a list of design constraints and analyzing and breaking codes that others have produced.

The fourth learning environment that we have developed supports students' work with geographic information systems. In one curriculum unit, called Excursions, students use Mapper software to design routes and create maps for touring through the waterfront and park areas of San Francisco. Since their tourist clients have certain constraints—such as touring by bike, wheelchair, or on foot—students need to understand slope, angles, and grade. In order to apply that understanding to map making, students also use spatial skills to learn about scale, representation, and graphing.

When teachers and students work in these environments, their leading activities are design projects. Mathematical concepts and methods are important resources for the work, but the activities are organized primarily as the design of buildings, models of population growth and decline, codes, or maps, rather than according to mathematical topics. Our curriculum, therefore, embeds the subject matter of mathematics in activities that are organized mainly by nonmathematical concepts and concerns. Thus, our curriculum represents an effort to realize Dewey's ([1916] 1966) proposals and goes quite far in the direction that Judd (1936) considered undesirable. We have been able to demonstrate, however, that growth of systematic mathematical understanding can be included in the activities of a curriculum that is primarily organized around activity that is meaningful in other terms.

With the teachers who collaborate with us in the design and development of our materials, we understand a major challenge of "uncovering the math" that is embedded in the students' design activities. (This contrasts with the challenge of "covering all the math" that faces teachers working with most other materials. The play on "covering" reminds some people that mathematical concepts and principles can be hidden under an overemphasis on symbolic manipulation.) To address this issue, we develop materials that are called mathematical activities, mathematical extensions, and mathematical investigations. These materials provide resources for teachers to develop mathematical topics systematically, using the design activities as examples and applications of the ideas. Examples of concepts that are treated in this way include measurement and scale, ratio and proportion, and functions. By relating these topics to design activities, we believe that students can better understand the concepts meaningfully, forming connections between mathematics and other topics. We particularly hope that the relations to design activities will decrease the proportion of students for whom mathematics is understood primarily as a set of arbitrary rules for manipulating symbolic expressions.

Some issues and preliminary findings.—We now report, very briefly, on four issues of theory and practice with which we have been concerned in this project. These issues illustrate ways in which practice-oriented concerns, at the level of assumptions or "first principles" (A. L. Brown 1994; Brown and Campione 1994; J. S. Brown 1991), and theory-oriented concerns about meanings of fundamental concepts and coherence of fundamental principles of cognition and learning converge.

Varieties of mathematical activity and understanding.—The question "What is mathematics?" is fundamental in the teaching and learning of mathematics. In traditional K–12 school mathematics education, mathematics is assumed to be a set of defined concepts, symbols and systems of representation, proved results, and valid computational procedures. Learning mathematics is assumed to be the acquisition of skill in the procedures and knowledge of the concepts' definitions and proved results that students can display by performing computations, reciting and explaining definitions, constructing proofs, and solving problems for which mathematical concepts and methods are relevant.

In the kind of mathematics education that we and others are trying to develop and understand, a different assumption about the nature of mathematical knowledge and learning is made. The assumption of this work, in the situative perspective, is that mathematics is a set of practices of inquiry and sense-making that include communication, questioning, understanding, explaining, and reasoning, and that learning mathematics is marked by increasing participation in an expanding range of such practices. Understanding mathematical knowing and learning as a set of practices of inquiry and sense-making presents fundamental theoretical questions for cognitive and social science, as well as major challenges for educational practice.

In MMAP, practices of inquiry and sense-making are focused on design projects, in which mathematical concepts and methods are often implicit in students' reasoning rather than being explicit topics. We have made some headway toward characterizing ways in which understandings of mathematical concepts and principles are involved in the practices that students and teachers are engaged in when they use MMAP curricula. One set of findings, by Beverly Bushéy (1997), involves contrasts between mathematical content that is predetermined in activity and mathematical content that is emergent. In standard mathematics homework exercises and tests, students' activities are designed to have specific mathematical content, and students perform series of small tasks, all of which use a specific mathematical procedure or concept. In MMAP projects, students' activities are organized differently. They work for an extended period (usually several weeks) on a project of designing something relatively complicated, such as a living and working space for a group of researchers. We designed these projects so that mathematical concepts and principles appropriate for middle school students emerge frequently, presenting themselves as both challenges and resources for students as they progress through their design work. These include a great deal of reasoning involving proportions, ratios, percentages, and basic functional relations among quantities. Although these topics are the constant fare of middle school mathematics, in the MMAP curriculum they emerge in the students' design activities as instrumental resources for understanding or communicating about the quantitative properties underlying their specific designs. For example, when students are laying out floor plans, they consider the proportional relations between lengths of lines in their diagrams and the physical lengths and widths of rooms that the diagrams represent, often taking meter sticks and finding physical distances that correspond to their drawings. The students are drawn to the activity of measuring the sizes of real spaces (e.g., finding the distance along the side of the classroom corresponds to a length of four meters) in relation to the scale they have set inside the ArchiTech' program because they want their designs to accurately meet the constraints of the design project. This is not a required activity and it is not one that students "take on" until they determine that it is necessary. In this sense, the issue of scale is emergent.

A particularly compelling contrast that Bushéy observed involved a student's work on a design problem and the same student's retrospective reaction to work on a traditional homework assignment. In the design problem, the student had participated in an effort by her work group to understand and represent the scale of a floor plan consistently and remarked, "Oh, God, this is complicated." In the homework, she had completed the assignment of converting a list of room sizes given in square feet to square yards, but had done that by dividing each number by three. The teacher, after explaining why the divisor was nine, remarked that the material was "a little more challenging," and the student said, "It didn't seem challenging until you told me I did it wrong." This example illustrates an important difference in which emergent mathematical reasoning can involve dealing with multiple constraints, while predetermined mathematical content specifies operations that have to be carried out repeatedly to get correct answers.

A related dimension of variation in the mathematical content of activity is the explicitness of reference to mathematical concepts and principles in discourse. Rick Berg et al. (1994) examined videotapes of students working on design projects in two different curriculum projects, the MMAP Antarctica curriculum and a unit called Design Spaces for People in the Seeing and Thinking Mathematically (STM) program of the Educational Development Corporation (Kleiman 1992). The STM activity involves forming a three-dimensional model using pieces of cardboard cut in the shapes of regular polygons. Berg et al. had expected to find a general tendency toward more explicit discussion of mathematical terms later in students' work as they moved into more analysis of designs they had created, and this seemed to occur in the MMAP curriculum. In working with the MMAP materials, students often took up general considerations about living arrangements early in their work, and dealt with issues such as room sizes in ordinal terms. An example of discourse early in the project was a comment by a student that the student's group should "widen this maybe just a little more and put the sinks in there. That's the other bathroom, it's identical sides." Later, when they had the general features of their design settled, they became concerned with some of the quantitative details, such as the exact numerical value of the scale of their drawing, including a discussion of whether they should set the scale at 1 cm : 120 cm (one and one-fifth meters) or 1 cm : 118 cm. On the other hand, in one group working with the STM curriculum, there was quite a bit of discussion early in the project involving general properties of geometric forms, particularly about shapes that could be laid out on a flat surface versus producing a surface that would be higher in the center (to cause water to run off), and discussion later turned more to whether the spaces they had designed would accommodate living requirements of the occupants. These examples illustrate different sequences in which explicit discussions of mathematical concepts and methods are related to situations in which the concepts and methods are used.

Engagement and visibility of participation by different students.—Activities in MMAP classrooms are focused mainly on small groups of students who work together to produce and analyze designs. This raises issues of differences in the ways that individual students participate in the group activities, and observation of their participation informs us about characteristics of the social organization of learning activity.

One aspect of this issue is whether different members of the group participate actively in the process. Concern is often expressed regarding the practical issue of whether group activities produce learning activities that are distributed unequally among the group, leading to unfair demands on some students or even more unequal learning than is true in standard classroom activity. To contribute to both the community of theory-oriented researchers and the community of practitioners that we participate in, it would be helpful to understand general principles of the distribution of activity among individuals who work collaboratively.

Not surprisingly, we have found that the phenomena are more complicated than would be expected from the version of the question that is common in discussion, that is, "Does group work produce unequal participation?" The ways in which individuals participate in collaborative learning activities is a complicated result of complicated factors, including their general patterns of participation in school and in the communities of students in which they are members. One example, discussed by Shelley Goldman and Raymond McDermott (1994), involved a student, Hector, who typically acted as if he was disengaged from participation in whole-class activities; for example, he uniformly did not hand in assigned work. On the first day of work on the Antarctica station, another student in his group had represented a bedroom that would be four meters by four meters, and the teacher asked whether that would be a reasonable size. Hector got a meter stick and measured four meters along a wall of the classroom and gestured to point out its size squared, providing the group with needed information about the semantic interpretation of their diagram. He, and the other members of the group, agreed that four meters by four meters would be "really big" for a bedroom, and the plan was adjusted. Later, during an activity of plotting a line graph relating insulation value to heating cost in their design, Hector participated in the process of rounding their data to match units on their graph and locating points on the graph. (This, in itself, was a fascinating and complex example of peer tutoring.) The period ended with Hector persuading another member of the group to keep Hector's graph in the other student's notebook. Despite his full participation and obvious success in the graphing task, he did not hand in his work.

Additional observations have confirmed this special version of participation. Middle school students sometimes shy away from whole-class interactions and accountability measures that might publicly confirm their interest. The design-based, group activities give students more leeway for their covert engagement with mathematical ideas and content.

A second issue of participation involves aspects of interaction that are often taken to indicate degrees of participation, but which our findings indicate are not. One such indicator is possession of the mouse at a computer terminal; the student who has the mouse is often assumed to be in a more directive position than the other students. Karen Cole (1994) examined the discourse of students at times when different students had the mouse and found that the situation is quite different. In fact, in these groups, the student with the mouse tended to make fewer directive comments and have fewer suggestions incorporated into the designs than other students in the group, probably because the student with the mouse was often occupied in carrying out suggestions provided by other students.

An issue that has not been understood well is how organizing students in small groups helps them come to participate in ever more complex mathematical practices. Methods of organizing student interaction, such as small group work, are often seen as static entities in which students are given a particular task to do and a particular role to play. If all goes well, the theory goes, students carry out the procedures they have been given.

Cole (1995) looked closely at how group engagement changed over the life of an MMAP design project, and found that students continue to organize or structure their interaction throughout the project, taking on new roles and practices, as they continually redefine the task. For example, she watched one group nominally working on the task, "Produce a floor plan on your computer." When they began, they interpreted the task as, "Copy our paper sketch onto the computer." As they interacted with the teacher, the computer program, and other groups, they saw that their paper design was flawed, and reinterpreted the task: "Produce a *correct* floor plan." Later, as they started to act more like designers, the task became, "Produce a correct, cost effective, and interesting floor plan."

This process was intimately tied to the way students interacted with mathematical concepts. As students interacted with their designs, each other, and the curriculum, they built three kinds of relationships with math concepts they encountered: definitional, operational, and social. So, for the concept of *metric measurement,* a definitional relationship that formed was, "a meter is the length of a meter stick." Students also formed operational relationships to the concept: "A meter is something you can use to describe the size of a room in a scale drawing." Finally, they formed social relationships with the concept: "A meter is something my partner's good at estimating." These relationships themselves continued to evolve throughout the life of the project in concert with the changing tasks and roles. For example, when the task was to produce a correct design, the skill of being able to estimate metric measurements became important for the group, and the group member who was good at it moved from a peripheral role to a more central role.

We conclude from these and other examples that the determinants of participation by students are subtle and complicated. Different students participate in group activities in different ways. Any characterization as simple as "more" and "less" participation will certainly miss crucial aspects of the participation and occasions for learning by different students in the group. In our judgment, there is a compelling need for progress in research that can provide a basis for characterizing varieties of participation by individuals in group learning activities in ways that inform both theories of learning in group participation as well as the design of activities for group learning and ways that teachers can influence those activities productively.

A third issue of participation involves assessment. Judit Moschkovich (1994) used concepts of ethnomathematics and the "didactical contract" between teachers and students to frame a set of principles of assessment of students' learning and accomplishments. The ethnomathematical perspective emphasizes that students should be assessed for their abilities to engage in mathematical practices that have legitimacy in activities that are significant in their present and future nonschool lives. Consideration of the didactical contract emphasizes that assessment practices should be a fair representation of students' participation and achievements in the activities that have importance in the classroom community. Student assessments in MMAP, developed jointly by MMAP teachers, researchers, and curriculum designers, include final project presentations and reports that parallel the practices of architectural design teams. These assessment activities can provide valuable occasions for the groups to synthesize their results and to participate in reflective analyses of their own and other groups' solutions to design problems. There are, however, many mathematical achievements by the students that are not shown in their class presentations and reports, but that occurred in the collaborative work in which the students engaged in developing their designs. The Middle-School Mathematics through Applications Project has begun to develop tools, techniques, and teaching practices that will support a more complete picture of students' achievements. This picture will need to include many aspects of authenticity, reflecting legitimate work practices as well as math activities important in the classroom community. One attractive possibility, which is now incorporated into the optional MMAP assessment activities, is for students to reflect on their mathematical activities and contribute to the record of their achievements through journal entries or a "mathematical mapping" of topics that MMAP activities include. Such practices, in which students can become active participants in their own assessments, can have the advantages of helping them develop standards of mathematical quality and responsibility for their own mathematical learning, as well as providing a more complete record of their work and learning.

Maintenance and persistence of coherent student activity.—We have given some attention to ways in which the activities of small groups of students are coherent and persistent. In traditional teaching practice, much attention is given to "classroom management," the work a teacher does to maintain an orderly activity structure in which students receive information and practice skills individually. The teacher is a kind of conductor who directs the class in activities that are more or less uniform. In classroom activities that are design based and distributed among small groups of students, direction cannot come from the single source of the teacher. In classrooms that we observed, there was an impressive degree of coherence in the students' activities that persisted across time. The question of how this happens is important both for the theory of social interaction and the design and practice of education in distributed activity.

Most of the time, the activity of a group of students is an interaction involving the students as agents interacting with technologies. We have begun to appreciate that the technologies that support this interaction are also diverse. They include the obvious technological resources of computers and calculators. They also include more ordinary technologies such as pencils and pieces of paper with grid marks for sketching plans, other sheets of paper that are set up for students to keep records of their progress in completing assignments, and meter sticks to use to measure lengths in the classroom or hallway that correspond to the lengths or widths of rooms that are represented in students' designs. We now realize that understanding engaged participation in the distributed learning activities of classrooms such as those we study requires us to consider the affordances of a large collection of technological resources that are available to support the students' work, including ordinary supplies as well as the more complex information technologies that have been developed recently.[1]

The ways in which teachers participate are also crucial, of course. Although we have only observed a few teachers working with the MMAP program, we have observed an interesting variety of ways that teachers influence student learning activities to facilitate their coherence and persistence. Teachers differ from each other in their characteristic patterns, and individual teachers differ in their patterns of interaction with students from class to class.

In one case, the teacher began by leading discussions of the general requirements of the design project and providing a group tutorial on the technology that included one member of each of the student working groups. As the students developed their projects, the teacher intervened very little in their substantive work except to respond to questions. In at least some of the class sessions, most of this teacher's interactions with students were to provide resources involving the various technologies that students could use in their work—for example, arranging for a group not working with a computer to get a file from their previous work printed by another group's computer, providing a piece of paper with a grid for a group to sketch their plan, or authorizing a student to get a meter stick from the room where supplies were kept.

Another teacher was considerably more directive regarding substantive aspects of her students' work, for example, assigning a task of constructing a graph based on data recorded from several designs. In one instance, this teacher had assigned a worksheet with questions about functional relationships between quantities in the design task, and some of her students asked insistently how they should answer the questions. The teacher's interactions with these students included the teacher examining what the student had already done, asking questions to arrive at an assessment of what the student already understood, and providing useful but incomplete help for the student to proceed. In one instance that we examined in some detail, the student returned for more help and the teacher gave increasingly directive assistance in response to the students' continued state of not completing the task to his satisfaction.

The difference between these two teachers' work is not simple. The first teacher was considerably more directive with another class that we observed, probably because she judged that those students needed a greater degree of structure to maintain productive and coherent activity.

The importance of these findings is in emphasizing that the maintenance of coherence and persistence in students' activity is an interaction involving the teacher, the students, and technological resources and the specific features of the tasks at hand. We cannot expect to design technologies that will engage students in meaningful activity without also considering the uses that teachers and students make of them. It is essential to focus on the design of the activities and social arrangements that support students' participation, and technological resources can contribute significantly as components of these designs, but technologies in themselves are not sufficient to support productive learning and teaching.

Organization of teachers' activities.—The changes in goals that are involved in reforming education imply profound changes in the practices of teaching. We discussed previously some ways in which teachers' activities influence and contribute to students' activities in learning. We also have addressed questions about how teachers' activities are organized in classrooms where materials like the MMAP program are used.

Jennifer Knudsen (1994) has begun developing an analytic framework for understanding practices of teaching that are consistent with general goals of educational reform at this time. Knudsen's proposal has grown out of her work with teachers in MMAP, including examination of videotapes of teaching activities and discussions with teachers about their practices. Empirical information for this proposal has also come from the contributions that MMAP teachers have made to the project's design and development of materials to be used with other teachers, in which they reflect their experiences in teaching the material, expressing their priorities and concerns regarding aspects of teaching with MMAP that are important or problematic.

Knudsen's framework distinguishes three dimensions that she calls *facets of teaching, planning and improvising,* and *concerns.* Facets of teaching include identifiable kinds of functional classroom activity such as guiding a group of students' design process, discussing relations between classroom activities and students' experiences out of school, facilitating the interactive work of student groups, and assessing and adjusting materials and methods to facilitate the progress of different groups of students. Planning and improvising are modes of teachers' work that reflect both the need for and the limitations of preparation for teaching when the activities of learning are distributed and open ended. Concerns are categories of awareness that function in teachers' work as broad organizing principles. These include fostering students' mathematical understanding, equity of students' access to and participation in mathematical activities, maintaining a balance between progress on general educational goals and progress in the detailed requirements of classroom activities, and reflection on and revision of their own teaching.

This scheme for thinking about teaching practices seems generally applicable to understanding the activities involved in successful teaching in reforming mathematics classrooms. Knudsen also identified a concern that is particularly salient for teachers using the MMAP program, which involves identifying the mathematical contents of students' activities and fostering mathematical conversations in which mathematical concepts and principles are significant and lead to improved student understanding. Knudsen considers this a concern, rather than a facet, because she has observed that its influence on teaching appears throughout teachers' activities, rather than constituting a facet that can be located in identifiable segments of activity.

Discussion.—The Middle-School Mathematics through Applications Project is an example of combined research and development of a kind that has been called a design experiment (Brown 1992; Collins 1992), pioneering research (Brown 1991), or interactive research and design (Stucky 1996), which combines the goals of developing resources for the reform of education and advancing fundamental understanding of learning, teaching, and cognition.

On the educational side, MMAP is exploring a version of an idea that plays a continuing role in discussions of educational practice and psychological theory—the idea that effective learning requires knowledge to be related to students' existing understanding and experience. Relative to most other mathematics curriculum projects, MMAP represents an extreme version of this idea. In most other projects, mathematical concepts and methods are related much more to general topics and activities than they are in traditional classroom instruction, but mathematical topics are always in focus and serve to organize the main issues that are discussed. In contrast, the issues that organize most of the discussion in MMAP classrooms are those that arise in the design of buildings, biological models, codes, or maps. Mathematics is in those discussions, but in relation to other issues. As we noted before, a continual challenge in the design of activities and in teaching is to *uncover* the mathematical contents of the activities; this contrasts with the major problem of traditional curriculum and teaching which is to *cover* as many mathematical topics as possible.

Exploring this idea of embedding mathematical content in other activities, including developing resources of information technology, plays a useful role in the general current efforts for educational reform. At the same time, the project addresses questions about fundamental social and cognitive processes of activity and learning. The general question that organizes our research is how learning occurs in the activities that teachers and students engage in with MMAP materials.

While we had general expectations about the character of activity and learning that we hoped would occur when we planned MMAP, most of the results of our research have an emergent quality. We observe and record interactive social behavior that occurs in classrooms, and take the challenge of making sense of it, trying out many possible conceptual frames, schemata, and theoretical ideas. The application of multiple conceptual frames is concrete in our activity, which emphasizes collaborative multidisciplinary analysis and interpretation of empirical material, especially videotape records. The aspects of theoretical issues on which we make progress, including the conceptual categories with which we formulate our interpretations and conclusions, grow out of an interaction of what is observed and the researchers' ideas, hunches, and areas of interest in school change, as well as the knowledge and practices that we bring from our various research disciplines.

In the next section, we argue that the situative perspective, as it is developing in cognitive- and social-science research, affords a subsumption of the psychological perspectives of behaviorism and cognitivity that can incorporate their strengths and values in a coherent framework. The results obtained thus far in MMAP seem to encourage that prospect. Our results, regarding maintenance of coherent and persistent activity structures, students' participation in the activities of learning, the organization of teachers' activities, and ways in which the contents of mathematical concepts are involved in different kinds of learning activity, are all concerned with both how people behave and how they use, interpret, and construct information. It is noteworthy that the behaviors that we mainly focus on are social interactions, in which the actions of several individuals mutually influence and reinforce or interfere with each other. Our behavioral focus, therefore, is more in line with the tradition of ethnography than of behavioristic psychology, but it is behavioral nonetheless.

More specifically, our research is enabling us to progress toward an understanding of thinking that includes processes of socially interactive behavior along with processes of transforming representations of information. Like others who have focused on social interactions in classrooms, our results are providing information about patterns of activity in the conceptual domains of subject matter disciplines. In their admittedly preliminary form, our results are beginning to provide understanding of ways in which the organization of activity shapes significant aspects of the conceptual contents of reasoning and learning. By considering the process of learning primarily from the point of view of students' participation in that activity, our results focus on the participation structures of learning activities. The situative perspective gives us a way to understand the effectiveness of group work and design tasks. We also attend closely to the contents of those activities, such as proportional reasoning and the concept of function, partly because our work is accountable to the subject-matter imperatives of mathematics education, and also because we want our results to advance fundamental cognitive science's understanding of processes of understanding and reasoning. It is easy to help students form definitional relationships with concepts; just give them the definition. Helping them form operational relationships requires a task on which to operate, and the more complex the task, the more potential there is for forming operational relationships and social relationships involving the concepts.

Relations between the Perspectives

Conceptualizations of Learning and Thinking

We consider the situative perspective as a general framework that can include the associationist/behaviorist and the structural/cognitive perspectives, rather than as an alternative that implies rejection of the other perspectives. If we consider thinking as an aspect of participation in social practice and ask how effective participation occurs, then factors of technical skill, strategic thinking, and conceptual understanding obviously are important.

We do not believe that the three perspectives compete at the level of making different predictions about thinking that can be tested in critical experiments. Whenever people think, their activity can be described in terms of their individual behavior and explained with hypotheses about stimulus-response associations. Their activity can also be described in terms of its consistency or inconsistency with accepted concepts and principles and explained with hypotheses about schemata and strategies. And their activity can be described in terms of social practices in which individuals coordinate their behavior with each other and with resources in the environment.

One possibility is to consider these perspectives simply as alternative ways of conceptualizing thinking and other activity, and to choose whichever one seems most useful for a given problem or question. On the other hand, it also seems valuable to consider whether one of the views may be more successful in providing a coherent and productive framework for studying and understanding the phenomena of thinking scientifically and for discussing alternative educational policies and practices.

The alternatives for developing a coherent general theory and discourse of practice are (1) to consider concepts about individual behavior and cognition as the basic explanatory level of the theory and to consider other people and environmental resources as contexts of behavior and cognition, or (2) to consider concepts about the behavior of larger systems that include individuals and systems in the environment as the basic explanatory level of the theory and to consider individual behavior and cognition as components of those systems, to be analyzed in relation to their contributions to system functioning. Clearly, current scientific knowledge and understanding does not provide a strong basis for deciding which of these alternatives is going to be more successful. We organize our work along the lines of the second alternative, and we believe that the topic of thinking provides a useful example topic in which to develop that alternative.

Educational Practices Aligned with the Theoretical Perspectives

As analytical perspectives, these views do not strictly imply different educational practices. Instead, by focusing attention on different aspects of learning and thinking, they provide conceptual frameworks that seem more natural for different learning arrangements or that seem to put different practices in favorable lights.

In a learning environment that emphasizes accumulation of skill in manipulating symbols, the students and teacher focus on students' success in learning to carry out procedures. The behaviorist perspective provides a natural focus on the students' acquisition of skill. At the same time, the cognitive perspective provides a valuable focus on the strategies of learning that students need to succeed. And, of course, the activities of learning and performance are situated practices, and analyses in the situative perspective show how practices of classroom discourse are organized for students to learn by listening to and watching the teacher, by completing problem sets, and by displaying their ability to perform correctly by working problems at the board and in tests.

In a learning environment that emphasizes conceptual understanding in subject matter disciplines, the students and teacher focus on students' success in discussing the accepted meanings of concepts and using concepts to solve problems and explain phenomena and procedures in a domain. The cognitive perspective provides a natural focus on the students' learning of schemata and thinking strategies that support this performance. Of course, these achievements can be characterized as behaviors; successful students can be characterized as having learned associations and responses at appropriate levels of generality. In the situative perspective, the practices of learning and performance in a concept-emphasizing environment differ from those of a skill-emphasizing environment, but they are still practices. Analyses in the situative perspective show how practices of classroom discourse are organized for students to learn by listening to and participating in discussions of concepts and constructing explanations that are judged by their correspondence with standard meanings and explanations.

In a learning environment that emphasizes participation in practices of inquiry, the students and teacher focus on students' activities of formulating and evaluating questions, problems, hypotheses, conjectures, evidence, examples, conclusions, arguments, and explanations in the work they do as individuals and in groups. The situative perspective provides a natural focus on the ways that students learn to contribute to the progress of inquiry in classroom discourse and how they progress as individuals in the development of their identities as knowers and learners. Of course, the students' skills, conceptual understanding, and strategies for learning and solving problems are crucial for their participation, and the behaviorist/associationist and domain-structural/cognitive perspectives are valuable in focusing on these aspects of students' growth in participation.

The behaviorist and cognitive perspectives on thinking present a dilemma. Behaviorism emphasizes actions involved in thinking, and cognitivity emphasizes conceptual structures and representations of information. These perspectives are not necessarily incompatible, but research and educational practice that have taken the two perspectives have addressed quite different questions about thinking. Behaviorists' questions are concerned with conditions in which different responses would be more or less likely, depending on factors such as motivation, reinforcement history, and time since their previous performance. Cognitivists' questions are concerned with the correspondence between students' conceptual understanding and accepted scientific or mathematical concepts and with the kinds of representations of information that are constructed and used in thinking, such as whether information used in the mind corresponds to rules or to models, and what kinds of representations support generalization of learned problem-solving procedures. In educational practice, behaviorists have been concerned with analyzing complex tasks into components and specifying instructional objectives and assessments that present and measure observable behaviors. Cognitivists have been concerned with arranging discussions in which students engage their intuitive understanding of phenomena or systems and develop new understandings through discourse and interaction with material and technological systems.

The situative perspective on thinking as socially organized practice can subsume the behaviorist and symbolic cognitive perspectives in an interesting and deep way. Rather than separating action from meaning, the situative perspective focuses on activities that communicate and construct meaning. This synthesizes the concerns of correct action and productive understanding. Rather than conceptualizing understanding of something as a private representation of it, constructed through transformations on mental symbols, understanding is viewed as the result of an activity that most typically involves people constructing common ground in conversation, using symbols that have meaning in their shared practice. Whether the activity is conducted by a group or by an individual, it involves an interaction of a person or people with symbolic material that has meaning through its function in communities that use it. In this view, operations on symbols are a form of practice, which members of a community use in their activities. Symbolic operations can function instrumentally in significant activities of the community, and correct performance of those operations is one of the ways in which members of the community participate in and contribute to the community's achievements.

In addition to subsuming behaviorist and symbolic cognitive views of thinking, the situative view also extends the framework for thinking to a set of issues that goes beyond the concerns of those perspectives. Educational research and development in the situative perspective is concerned with how students can acquire practices of discourse and inquiry in which meaning is constructed and shared. This goes beyond issues of providing students with acquired skills in manipulating symbols and of understanding of the symbols' conceptual meaning, to include abilities to participate in practices of formulating and evaluating questions and problems, hypotheses and conjectures, evidence and examples, and conclusions and arguments. In behaviorist and cognitive views, these processes have generally been considered as means to the end of knowledge and understanding of content. In the situative view, they are themselves goals of learning—perhaps the central goals.

A Cultural Icon of Thinking

Our society's most familiar image of thinking is Rodin's sculpture *The Thinker.* The meaning of thinking conveyed by that image has been stated aptly by Albert Elsen.

> Alone in Rodin's *Gates of Hell, The Thinker,* who represents the worker-artist-poet, demonstrates the good of intellect. Because in society he alone is thoughtful, the artist can detach himself from the crowd and its passions in order to reveal humanity to itself. *The Thinker* has replaced God and Christ in the supreme juror's seat, and what is around him is not his sentence for humanity but rather what he judges to be the human condition: one of ceaseless, futile striving.
>
> To understand more fully *The Thinker* as the creator and what is required of the creative artist, look again at the sculpture. By placing his right hand against his mouth, *The Thinker* forecloses speech. His lack of irises denies the man sight. Although the ears are not covered, he hears nothing but the soundings of his thought. Without a scrotum he cannot reproduce himself biologically, only through art. It was no coincidence that Rodin shut off the figure's ability to perceive sensation. These decisions, coupled with the overall pose, compel *The Thinker*'s complete intellectual absorption. He is an image of the sensory isolation required of the most intense creative thinking.
>
> Despite the physicality of the man, the sculpture's focus is upon the brain as the center of creation. Hard thought must come before and after the physical work of art. To win that focus, Rodin bent the head forward and down and shaped the hair into a domical form that attracts light to it. (Elsen 1985, pp. 65–71)

Elsen's discussion expresses a common view of thinking as intense, private activity, occurring in isolation from the world and other people. Of course, there are circumstances in which productive thinking can or must occur in that way. In its original setting, *The Thinker* is seated near the top of *The Gates of Hell,* surrounded by the sights and sounds of eternal torment.[2] No wonder he finds it unspeakable, and his inability to see (or hear?) the agony around him is surely a blessing.

The image of thinking conveyed by Rodin's statue fits well within the associationist/behaviorist and domain-structural/cognitive perspectives, at least from Rodin's romanticist artistic world view. *The Thinker* is beset by a problem for which he has no solution in his immediately available repertoire. In his struggle to find a better idea, he shuts himself off from the stimuli to which his inadequate thoughts are associated. The behaviorist advice to cast a larger mental net and the cognitive strategy of constructing a mental representation from a general schema are consistent with a remark of Rodin's in a letter: "I conceived [a] thinker, a naked man, seated upon a rock, his feet drawn under him, his fist against his teeth, he dreams. The fertile thought slowly elaborates itself within his brain. He is no longer dreamer, he is creator" (Elsen 1985, p. 43).

Of course, we appreciate Rodin's remarkable artistic achievement, but we believe that the image that it gives thinking is unfortunate. We wish that *The Thinker* had a different name. When we see it (which we do quite often; one of its sites is on the Stanford campus between the two buildings that house the School of Education), we like to think of it as *The Mourner.*

We prefer a different image of thinking than that conveyed by *The Thinker.* We would prefer icons that represent groups of people in animated conversations, interacting with some material that they are reasoning about and with which they are developing representations of their ideas. Raphael's *The School of Athens* seems more apt to us as an icon for thinking than *The Thinker,* although the dynamic quality of thinking is difficult to capture in a static form, so a representation that is extended dynamically in time would be even more appropriate.

There are many examples of the kind of event that we consider prototypical of thinking. One extended example was described by James Gleick (1987), in a chapter of his book *Chaos* entitled "The Dynamical Systems Collective." A few young physics graduate students at the University of California, Santa Cruz, in the late 1970s became interested in the mathematics of nonlinear dynamical systems and worked individually and together at the intellectual and social margins of their department and field. Their work eventually provided a significant part of what is recognized now as a major field of scientific and mathematical research. The story includes persistence by the group members in their unorthodoxy, as well as a few important items of encouragement and recognition from two local faculty members and from the scientific community outside of their local department. For a considerable time, the group mainly published its work in coauthored papers. In Gleick's words:

> By now the collective was meeting regularly in an outsized old Santa Cruz house not far from the beach. The house accumulated flea-market furniture and computer equipment, much of which was devoted to the roulette problem. Shaw kept a piano there, on which he would play baroque music or improvise his own blend of the classical and modern. In their meetings the physicists developed a working style, a routine of throwing out ideas and filtering them through some sieve of practicality, reading the literature, and conceiving papers of their own. Eventually they learned to collaborate on journal articles in a reasonably efficient round-robin way, but the first paper was Shaw's, one of the few he would produce, and he kept the writing of it to himself, characteristically. (Gleick 1987, p. 258)

In this image, thinking occurs in the activity of groups of people who work on a collection of related problems over an extended time. The thinking activities of such groups are a complicated combination of things they do together, such as discussing theoretical problems, as well as things they do individually, such as writing some papers. They also involve a complex interaction with larger communities in which they both resist and depend on the ideas and practices of their mainstream colleagues.

We would like to see a shift in the society's conception of thinking toward the flexible, socially interactive activity exemplified by Gleick, rather than the image of Rodin's statue. We see many versions of that image of thinking, involving interactions among people and between people and technological resources, on the videotapes of students and teachers working with the MMAP program and in the materials of other educational programs that emphasize students' participation in inquiry and their development of identities as learners and knowers. We hope that these are harbingers of learning practices that will become increasingly prevalent. We also are encouraged by the progress that we and others are making toward a scientific understanding of thinking and learning to think, especially using the situative perspective. Together, we believe these developments provide productive advances in the theory and practice of thinking and learning to think that are valuable for both science and society.

Notes

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1. The term "affordance" was coined by James Gibson (1986; also see Reed 1996) to refer to the properties of objects and systems in environments that support activities. The idea focuses on the interactive aspects of activity, in which people, material systems, and information technologies all contribute to the results that are achieved.

2. As Elsen (1985) related, Rodin included *The Thinker* in his plans for *The Gates of Hell*, which was commissioned in 1880 by the French government, and Rodin sculpted a 27-inch clay model of *The Thinker* that he showed privately

and in exhibitions. The government, however, did not call for delivery of *The Gates of Hell,* and a bronze case of it was not made during Rodin's lifetime. Rodin sculpted the enlarged version of *The Thinker* from 1901 to 1904. Regarding *The Gates of Hell,* Rodin wrote in 1904 that he had initially planned a statue of Dante, the thinker who created *The Divine Comedy,* from which Rodin took his theme. The plan for including Dante was not satisfactory, however, so Rodin substituted the figure that has become our culture's icon for thinking. One of the sites of *The Thinker* is, by Rodin's instruction, at the graves of Rodin and his wife. "A photograph taken during the 1917 funeral service shows *The Thinker* in a new context, poised on the edge of the open grave, as if looking into it at his maker, evoking still another ancient message: art is long, life is brief. . . . It was as if Rodin waited until the end of his life to leave us his last word on *The Thinker*'s identity. Directly beneath the sculpture, inscribed in large capital letters on the front of the pedestal, is a single word that identifies both the statue and the artist: RODIN" (Elsen 1985, pp. 140–41).

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