This week

Please hand in HW3
Lab report Lab 2 due next week

Today we continue last 2
Polarization

Imagine a wave traveling in the z direction. A possible solution for the E field is:

$$\vec{E} = E_0 \sin(kz - \omega t) \hat{x}$$

We call such a wave linearly polarized in the x direction.
Polarization

However, a more general solution is:

\[ \hat{E} = E_x \sin(kz - \omega t)\hat{x} + E_y \sin(kz - \omega t + \varphi)\hat{y} \]

\( E_x \neq E_y \) but more important is \( \varphi \)

elliptically polarized light
(draw on board)
special cases:
\( E_x = E_y \) circularly polarized light
\( \varphi = 0 \) linearly polarized light.
unpolarized light: direction of \( E \) changes randomly with time and position in space
usually due to fluctuations in the source
(either in space or time)
Can be used to measure and change the polarization of the light

“dichroic polarizer” absorbs light with E parallel to a specific direction.

Electrons in the material are freer to move in 1 direction than another. The component of E perpendicular to the electron motion is transmitted.
Waves, reflection, and refraction

Let's derive the reflection law, and Snell's law, in a more fundamental way. We'll also get more information as well about the reflected and refracted waves. We'll see also how these things affect polarization.

Imagine a plane wave incident on the interface between 2 media so that the poynting vector (direction indicated by blue line) makes an angle \( \theta \) rst the interface's perpendicular

- we know \( \vec{E} \) is \( \perp \) to \( \vec{S} \)
- We can see immediate that there is something special about the y direction. If the E field is in the y direction, it is parallel to the interface

\[
\vec{E} = E_y \hat{y} + E_{y\perp} \hat{y}_{\perp}
\]

When \( \vec{E} \parallel \hat{y} \) \( \rightarrow \) transverse electric
If \( \vec{E} \parallel \hat{y}_{\perp} \) then \( \vec{B} \) is along \( \hat{y} \) \( \rightarrow \) transverse magnetic
Any arbitrary \( \vec{E} \) can be written as a sum of TE and TM
Want to understand how \( E_r \) and \( E_t \) are related to \( E \)

\[
\vec{E} = E_0 \hat{y} e^{i(k \cdot \vec{r} - \omega t)}
\]

at interface \(( \pm \varepsilon )\)

\( \vec{E} + \vec{E}_r = \vec{E}_t \)

if we want this relation to hold at all times

\( \omega = \omega_r = \omega_t \)

if we want it to be true at any value \( x \)-\( y \) at a fixed time \( t \)

\( \vec{k} \cdot \vec{r} = \vec{k}_r \cdot \vec{r} = \vec{k}_t \cdot \vec{r} \)
\[ \vec{k} \cdot \vec{r} = -|\vec{k}| |\vec{r}| \sin \theta_i \]

\[ \vec{k} \cdot \vec{r} = \vec{k}_r \cdot \vec{r} \]

\[ |\vec{k}| \sin \theta_i = |\vec{k}_r| \sin \theta_i \]

\[ |\vec{k}| = |\vec{k}_r| \] since in the same media, so

\[ \theta_i = \theta_r \]

\[ v = \frac{\omega}{k} \]
\[ \vec{k}_r \cdot \vec{r} = \vec{k}_t \cdot \vec{r} \]
\[ |\vec{k}_r| \sin \theta_r = |\vec{k}_t| \sin \theta_t \]
\[ \frac{\omega}{k} = \nu \quad \text{so} \quad k = \frac{n}{c} \omega \]
\[ \frac{\omega_r n_r}{c} \sin \theta_r = \frac{\omega_t n_t}{c} \sin \theta_t \]
\[ n_r \sin \theta_r = n_t \sin \theta_t \]

General property of waves. We did not need Maxwell’s equations to get this. So true for sound waves, etc. (if wave length shortens, for matching conditions, light must bend)
TE

What are the relative sizes of $|\vec{E}|$, $|\vec{E}_r|$, and $|\vec{E}_t|$?

Use that Maxwell's equations require continuity of components of $E$ and $B$ parallel to interface be continuous. and to get relation between $|\vec{E}|$ and $|\vec{B}|$

$$\vec{E} = E_0 \hat{y} \quad \vec{E}_r = E_r \hat{y} \quad \vec{E}_t = E_t \hat{y}$$

$$E_0 + E_r = E_t$$

$$\vec{B} = B_0 \cos \theta \hat{x} - B_0 \sin \theta \hat{z}$$

$$\vec{B}_r = -B_r \cos \theta \hat{x} - B_r \sin \theta \hat{z}$$

$$\vec{B}_t = -B_t \cos \theta \hat{x} - B_t \sin \theta \hat{z}$$

Demand continuity of B field parallel to interface $B \cos \theta - B_r \cos \theta = B_t \cos \theta_t$

since $E = vB = \frac{c}{n} B$

$$n_1 E_0 \cos \theta - n_1 E_r \cos \theta = n_2 E_t \cos \theta_t$$

Put together and define $n \equiv \frac{n_2}{n_1}$

$$\frac{E_r}{E_0} = \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}}$$

$$\frac{E_r}{E_0} = \frac{2 \cos \theta}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}}$$

(Similar formula for TM mode: see text)
Polarization and reflection

\[ r_{TE} = \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} \]

\[ r_{TM} = \frac{-n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \]

\[ n = \frac{n_2}{n_1} \]

Pure TE at this angle
(light reflecting off water)

\[ \theta = A \tan \left( \frac{n_2}{n_1} \right) \]

Brewster's Angle

for air/water

\[ \theta = \tan \left( \frac{1.3}{1} \right) = 53^\circ \]
Hints

See last week’s lecture for lab 2 hints
\[E_0 + E_r = E_T\]
\[n_1 E_0 \cos \theta - n_1 E_r \cos \theta = n_2 E_r \cos \theta_T\]

\[E_r = E_T - E_0\]
\[n = \frac{n_2}{n_1}\]
\[E_r = n^{-1} \left( \frac{\cos \theta}{\cos \theta_T} - n^{-1} E_r \frac{\cos \theta}{E_0 \cos \theta_T} - 1 \right)\]
\[E_r \left( 1 + n^{-1} \frac{\cos \theta}{\cos \theta_T} \right) = n^{-1} \frac{\cos \theta}{\cos \theta_T} - 1\]
\[E_r = \frac{\cos \theta - n \cos \theta_T}{\cos \theta + n \cos \theta_T}\]
\[n_1 \sin \theta = n_2 \sin \theta_T\]
\[\sin \theta = n \sin \theta_T\]
\[n \cos \theta_T = n \sqrt{1 - \sin^2 \theta_T} = \sqrt{n^2 - n^2 \sin^2 \theta_T} = \sqrt{n^2 - \sin^2 \theta}\]
\[E_r = \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}}\]