Review: Michelson Interferometer

\[ I = 4I_0 \cos^2 \left( \frac{\delta}{2} \right) \quad \delta = k(s_1 - s_2) + (\phi_1 - \phi_2) \]

Michelson: \[ \delta = 2k(x_1 - x_2) \] \( x_1, x_2 \) are distances to mirrors 1+2

constructive interference \[ \frac{2(x_1 - x_2)}{\lambda} = m \]

destructive interference \[ \frac{2(x_1 - x_2)}{\lambda} = m + \frac{1}{2} \]

1. Measure average wavelength:

\[ \Delta m = \frac{2A_{x_1}}{\lambda_{av}} \]

\[ \lambda_{av} = \frac{2v \Delta x}{T} \]

\[ T = \frac{2v \Delta x}{\lambda_{av}} \]

ex: \( T \) fringes in time \( T \)

\[ \Delta x_1 = \sqrt{2} \quad \Delta m = 7 \]

\[ T = \frac{2v \sqrt{2}}{\lambda_{av}} \Rightarrow \lambda_{av} = \frac{2v \sqrt{2}}{7} \]
Sodium lamp: $\lambda_1, \lambda_2$

$$\Delta \lambda = \lambda_1 - \lambda_2 = \frac{\lambda_0^2}{2\Delta x_1}$$

$$\Delta x_1 = \sqrt{2}$$

$\tau$ is the time between maxima of beats

Coherence

Envelope function

Gaussian?

$$A \exp\left(-\frac{(t-t_0)^2}{\tau^2}\right)$$

$\tau$ is not coherence time!

Time to move mirror such that path length difference is one coherence length

$$l_t = 2V_{\text{mirror}} \tau$$

Path length is twice mirror distance
Single-slit diffraction

$P$ is distant enough that rays are parallel.

Wave fronts

$s = +b/2$

$s = 0$

$s = -b/2$

$r_0$ is distance to $P$ from point @ $s = 0$

$r_0 + \Delta$ is distance @ any $s$:

$\Delta = s \sin \theta$
Infinitesimal E field at \( P \) due to wave at \( S \):

\[
dE_p = \left( \frac{E_L}{r_o + \Delta} \right) e^{i(k(r_o + \Delta) - \omega t)}
\]

\[
\approx \left( \frac{E_L}{r_o} \right) e^{i(kr_o - \omega t)} e^{ik\Delta}
\]

\[
\Delta \approx r_o \quad \text{keep } \Delta \text{ in phase}
\]

\( E_L \) is field on left side

\( \frac{1}{r_o + \Delta} \) is inverse dependence of field from point: \( \Delta \to \frac{1}{r^2} \Rightarrow E \propto \frac{1}{r} \)

\[
F_p = \int dE_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \int_{-b/2}^{b/2} ds e^{ik\sin \theta}
\]

\[
= \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \left( \frac{e^{ikbsin\theta} - e^{-ikbsin\theta}}{ik\sin \theta} \right)
\]

\[
\beta \equiv \frac{kb\sin \theta}{2} \Rightarrow \quad F_p = \frac{E_L b}{r_o} e^{i(kr_o - \omega t)} \left( \frac{\sin \beta}{r^3} \right)
\]
Irradiance:

\[ I = \frac{\varepsilon_0 c}{2} \left( \frac{E_L b}{r_0} \right)^2 \left( \frac{\sin^2 \beta}{\beta^2} \right) \]

\[ I = I_0 \left( \frac{\sin^2 \beta}{\beta^2} \right) \]

\[ I = 0 \quad \text{at} \quad \beta = m\pi \quad m = \pm 1, \pm 2, \pm 3, \ldots, \quad \text{not} \quad \beta = 0! \]
\[ \frac{1}{2} k b \sin \Theta = m \pi \]

\[ \frac{b}{b} \sin \Theta = m \]

Zeros:

\[ m \lambda = b \sin \Theta \]

\[ m = \pm 1, \pm 2, \pm 3, \ldots \]

On a screen:

\[ \sin \Theta = \tan \Theta \approx \frac{y}{L} \]

Zeros:

\[ m \lambda = b \frac{y}{L} \]

\[ y_m = \frac{m \lambda L}{b} \]

\[ m = \pm 1, \pm 2, \pm 3, \ldots \]
Width of central maximum:

\[-\frac{\lambda}{b} \leq \sin \theta \leq \frac{\lambda}{b}\]

\[\Delta (\sin \theta) \approx \Delta \theta = \frac{2\lambda}{b}\]

On screen: \[\Delta y = \frac{2\lambda L}{b}\]

Note: width of maximum is inversely proportional to slit width!

(width of bright line on screen is \(b\))