Interference

$s_1, s_2$ are distances traveled from point at which phase is known.

Electric field @ point $P$: $\vec{E}_p = \vec{E}_1 + \vec{E}_2$

(assume polarization is same) $E_p = E_1 + E_2$
Irradiance \( \propto |E|^2 \propto E_1 E_1^* = (E_1 + E_2)(E_1^* + E_2^*) \)
\[ = E_1 E_1^* + E_2 E_2^* + E_1 E_2^* + E_1^* E_2 \]
\[ I_p = I_1 + I_2 + \text{"interference term"} \]

What is interference term?

\[ E_1 = E_{1 \max} e^{i(ks_1 + \omega t + \phi_1)} \]
\[ E_2 = E_{2 \max} e^{i(ks_2 + \omega t + \phi_2)} \]

\( \phi_1, \phi_2 \) are initial phases @ 1, 2

\( s_1, s_2 \) are distances traveled from 1, 2

\[ E_1 E_2^* + E_1^* E_2 = \]
\[ E_{1 \max} E_{2 \max} \left[ e^{i(ks_1 + \omega t + \phi_1)} - e^{i(ks_2 + \omega t + \phi_2)} + e^{-i(ks_1 + \omega t + \phi_1)} - e^{-i(ks_2 + \omega t + \phi_2)} \right] \]
\[ = 2E_{1 \max} E_{2 \max} \cos(\delta) \]
\[ \delta = k(s_1 - s_2) + (\phi_1 - \phi_2) \]
So:

\[ I_\rho = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos(\delta) \]

interference term

Special case:

\[ I_1 = I_2 = I_0 \]

\[ I_\rho = 2I_0 + 2I_0 \cos(\delta) = 2I_0 (1 + \cos(\delta)) = 4I_0 \cos^2(\delta/2) \]

\[ I_{\rho_{\text{max}}} = 4I_0 \] (twice intensity without interference)

\[ I_{\rho_{\text{min}}} = 0 \]
Example: Young's Double Slit

\[ \delta = k_s(s_1 - s_2) + (\phi_1 - \phi_2) \]

\[ s_1 - s_2 = \Delta = a \sin \theta \]

\[ \phi_1 - \phi_2 = 0 \quad \text{(wave fronts hit slits at same time)} \]

\[ \therefore \delta = k a \sin \theta \]

\[ I = 4I_0 \cos^2 \left( \frac{\delta}{2} \right) = 4I_0 \cos^2 \left( \frac{k a \sin \theta}{2} \right) \]

\[ = 4I_0 \cos^2 \left( \frac{\pi a \sin \theta}{\lambda} \right) \]
Coherence

No source is perfectly monochromatic. It may be monochromatic over time \( \tau_0 \):

\[ \tau_0 = \langle \tau_2 \rangle \]

"coherence time"

Fourier transform of \( f(t) \) is:

\[ |g(\omega)|^2 \]

FWHM = \( \frac{2\pi}{\tau_0} \)

\( \Delta \omega = \frac{2\pi}{\tau_0} \)
Note: \( \Delta \omega \Delta \omega = 2\pi \)  \( E = \hbar \omega \Rightarrow \Delta E \Delta \omega = 2\pi \hbar = h \)

**Coherence Length**

\[
l_t = c \tau_0 = \frac{c \cdot 2\pi}{\Delta \omega} = \frac{c}{\Delta \nu} \quad \Delta \nu = \frac{\Delta \omega}{2\pi}
\]

(length in space of coherent wave train)

"Line width"

\[
\gamma = \frac{c}{\lambda} \quad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2} \quad \Rightarrow \quad \Delta \nu = -\frac{c \Delta \lambda}{\lambda^2}
\]

\[
l_t = \frac{c}{\Delta \nu} = \frac{\lambda^2}{\Delta \lambda}
\]

\[
\Delta \lambda = \frac{\lambda^2}{l_t}
\]

In spectroscopy, common to measure \( \lambda, \Delta \lambda \) is "line width" of a source with wavelength \( \lambda \)
Conditions for observing interference

\[ I = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos (\delta) \]
\[ S = k_s (s_1 - s_2) + (\phi_1 - \phi_2) \]

\[ \text{Need to interfere beams emitted within } t = \tau_0 \text{ of each other: } |s_1 - s_2| < \frac{\lambda}{4} \]

otherwise \( S \) not constant (rapidly varying on scale of \( \tau_0 \))

White light: \( \lambda_t = \frac{\lambda^2}{\Delta\lambda} \approx \frac{(500 \text{ nm})^2}{300 \text{ nm}} \approx 800 \text{ nm} \approx 2\lambda ! \)

Gas discharge lamp: \( \lambda_t \approx 1 \text{ mm} - 1 \text{ m} \)

He-Ne laser: \( \lambda_t \) up to 10 km !