Last time:

Diffraction grating:

\[ \alpha (\alpha + x) \theta_p \]

Maxima occur at:

\[ \rho \lambda = \alpha (\sin \theta_i + \sin \theta_p) \]

\[ \rho = 0, 1, 2, \ldots \]
Bohr model of Hydrogen

\[ \frac{mv^2}{r} = \frac{ke^2}{r^2} \]
\[ v = \sqrt{\frac{ke^2}{mr}} \]

For \( \frac{1}{r} \) potential:
\[ K = -\frac{U}{2} \]
\[ E = K + U = +\frac{U}{2} < 0 \]

\[ E = -\frac{1}{2} mv^2 = -\frac{ke^2}{2r} \]

Bohr: hypothesized angular momentum was quantized:

\[ L = n\hbar = mvr = \sqrt{\frac{ke^2}{mr}} \]

Actually, Bohr assumed \( \Delta E_{\text{emitted}} = E_2 - E_1 = h\nu \)

and \[ \nu = \frac{1}{r} \]
\[ L = n \hbar = \sqrt{\frac{\hbar}{m}} \]

\[ \Gamma_n = n^2 \frac{\hbar^2}{2 \hbar m} \]

\[ E_n = \frac{1}{2} U_n = -\frac{k^2}{2r_n} = -\frac{1}{n^2} \left( \frac{k^2}{\hbar^2} \right) m = -\frac{RE}{\hbar^2} \]

\[ R_E = \frac{k^2 e^4 m}{2 \hbar^2} \]

\[ R_E \approx 13.6 \text{ eV} \]

Light emission at energies:

\[ \Delta E = E_{n_f} - E_{n_i} = R_E \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad n_f < n_i \]

\[ \lambda = \frac{hc}{\Delta E} \implies \frac{1}{\lambda} = \frac{R_E}{hc} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ \frac{R_E}{hc} = R_\infty = \text{"Rydberg"} \approx 1.097 \times 10^7 \text{ m}^{-1} \]

\[ R_\infty = 1.0973731568525 \times 10^7 \text{ m}^{-1} \]

(very well known constant!)
Finite mass of nucleus $m_p$:

$$M_e \Rightarrow \frac{M_e m_p}{M_e + m_p} \quad \text{(reduced mass)}$$

$$R_H = \frac{R \infty}{1 + \frac{M_e}{m_p}} \approx 1.0967758341 \times 10^{-7} \text{ m}^{-1}$$

Fun facts:

$R_1$ is "Bohr radius" $= \frac{\hbar^2}{k^2 e^2 m_e} = 5.29 \times 10^{-11} \text{ m}$

$E_n = -\frac{Z^2}{n^2} R_E$  

If $Z > \frac{1}{\alpha} \approx 137$, $E_n \sim m_e c^2$ and so $Z = 137$ nucleus can create a bound electron and eject a positron "nuclear collapse".
\[ \frac{1}{\lambda_{ni}} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad E_{ni} = 13.6eV \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

Various series of lines named according to \( n_f \):

\( n_f = 1 \) : Lyman series  \( E_{ni} = 13.6eV \left( 1 - \frac{1}{n_i^2} \right) > 10.2eV \) 
(all in ultraviolet)

\( n_f = 2 \) : Balmer series  \( E_{ni} = 13.6eV \left( \frac{1}{4} - \frac{1}{n_i^2} \right) \)  \( n_i \geq 3 \)

<table>
<thead>
<tr>
<th>( n_i )</th>
<th>( \lambda ) (nm)</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>656</td>
<td>red</td>
</tr>
<tr>
<td>4</td>
<td>486</td>
<td>cyan</td>
</tr>
<tr>
<td>5</td>
<td>434</td>
<td>violet</td>
</tr>
<tr>
<td>6</td>
<td>410</td>
<td>violet</td>
</tr>
<tr>
<td>7</td>
<td>397</td>
<td>probably not visible to eye</td>
</tr>
</tbody>
</table>

\( n_f = 3 \) : Paschen  \( (\lambda \geq 820\,nm) \)  IR

\( n_f = 4 \) : Brackett  \( (\lambda \geq 1460\,nm) \)  IR

etc.
Tips for Lab 6:

1. Use Hg spectrum to calibrate spectrometer
   a. Take data for \( p = -2, -1, +1, +2 \) (at least) for as many lines as you can see
   b. Plot \( \sin \theta_p \) vs. \( p \) (no error in \( p \))
      fit line to get slope \( \frac{1}{a} \)
2. Measure sodium doublet. Take data for \( p = -2, -1, +1, +2 \)
   for each of 2 lines.
   a. For each line, make plot of \( \sin \theta_p \) vs. \( p \) slope = \( \lambda/a \)
   b. Determine \( \lambda_{av} \pm \sigma_{\lambda_{av}} \) and \( \Delta \lambda \) and compare to Lab 4
3. Measure \( \lambda \)'s for Balmer series.
   \( \frac{1}{\lambda} = R_H \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \)
   plot \( \frac{1}{\lambda} \) vs. \( \frac{1}{n^2} \) to get \( R_H \)