EXAM#2 —Phy273—Fall 2001
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1 page, 4 problems, 70 points

Partial credit will be given for correct work shown. Also, if you miss an earlier part of a problem do not give up on later parts of the problem, even if they require the result of the earlier part. You can get partial credit by just solving the later part in a more general way. **Reminder: (i) Use dimensional analysis to check your work. (ii) Ask yourself if your answers are reasonable: correct sign, real number for real quantity, sensible magnitude, limits, etc. (iii) Use your knowledge from the course. Don't try to solve everything from scratch.**

1. A long uniform string of mass density 0.1 kg/m is stretched with a force of 50 N. One end of the string ($x = 0$) is oscillated transversely (sinusoidally) with an amplitude of 0.02 m and a period of 0.1 sec, so that travelling waves in the $+x$ direction are set up. (a) What is the velocity of the waves? (b) What is their wavelength? (c) If at the driving end ($x = 0$) the displacement y at $t = 0$ is 0.01 m with $\partial y/\partial t$ negative, what is the function $y(x, t)$ describing the resulting travelling waves, with x in meters and t in seconds? [5+5+10=20 *points*]

2. In a homework problem you calculated the fraction of sound energy that enters the water when plane sound waves are incident from the air, by matching the waves across the air-water interface. (The answer was 0.11 %.) [10+5=15 *points*]

- **(a)** State precisely and explain the physical basis for all of the boundary conditions for matching sound waves across the interface. Express your answer in terms of the sound displacement fields $s_{1,2}(x, t)$ and bulk moduli $K_{1,2}$ in the air and water, with the interface at $x = 0$.
- **(b)** Suppose the incident wave has displacement $s_{in}(x, t) = f(x - vt)$ for some function f, where x is the coordinate normal to the interface. Neglecting the weak transmitted wave, find the reflected wave displacement $s_{ref}(x, t)$ in terms of the function f.

3. The sun emits radiation that has an intensity of 1380 W/m^2 just outside the earth's atmosphere. The earth is 150 million kilometers from the sun. (a) What is the total power output of the sun? (b) What would be the intensity of solar radiation viewed from a distance of 100 light years? (1 y $\simeq 3.16 \times 10^7$ s $\simeq \pi \times 10^7$ s.) (c) How much solar radiation energy is contained at any moment in a cubic meter just outside the earth's atmosphere? (Neglect reflection off the atmosphere.) [5+5+5=15 *points*]

4. The wave equation for waves on a string with some stiffness (resistance to bending) has the form

$$\frac{\partial^2 y}{\partial t^2} = a\frac{\partial^2 y}{\partial x^2} - b\frac{\partial^4 y}{\partial x^4}, \tag{1}$$

with a and b both positive. The term with coefficient a is the tension term, while that with coefficient b is the stiffness term. [2+5+5+3+5=20 *points*]

- **(a)** What are the dimensions of a and b? (I.e. what powers of length, time and mass do they contain?)
- **(b)** Find the dispersion relation $\omega = \omega(k)$ for waves satisfying this equation.
- **(c)** Sketch the dispersion relation. Does the stiffness term make the phase and group velocities smaller, larger, or the same as without this term? Give a physical reason for this.
- **(d)** What *dimensionless* quantity involving the wavenumber k should be much less than unity if (i) the stiffness term is negligible compared with the tension term in the dispersion relation, or (ii) the tension term is negligible compared with the stiffness term?
- **(e)** What are the phase and group velocities under the conditions (i) and (ii) of the previous part?