HW#6H —Phy273—Fall 2001 Prof. Ted Jacobson
Due before class, Monday, Oct. 15, 2000 Room 4117, (301)405-6020
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Normal modes of a loaded guitar string

If a small piece of masking tape is placed at the middle of a guitar string, and the string is plucked, one clearly hears two separate notes, the fundamental mode of the string and the first harmonic. Listening more closely, one notices a beat frequency. The first harmonic is unaffected by the tape, since the midpoint is anyway at a node, but the fundamental is slightly lower in frequency than without the tape, since the tape adds inertia to this mode. Find the change in frequency due to the presence of the tape.

To set up this calculation, suppose the string has tension F, mass density μ, and length L, and treat the tape as a point mass m. One way to proceed is to write the general solution to the wave equation for each half of the string, and then find the combination of these solutions that meets the boundary conditions: (i) fixed string at the two endpoints, and (ii) the tape mass point satisfies Newton's equation $F_y = m\ddot{y}$. Condition (ii) will determine the jump on the slope of the string from one side of the mass point to the other. This jump will produce the finite (as opposed to infinitesimal) force needed to satisfy Newton's equation for the finite mass m.

1. Do the experiment.

2. Find an exact equation which determines the frequencies of the harmonics. Express your result in terms of the ratio m/M, where $M = \mu L$ is the total mass of the unloaded string. The solution to the problem is a transcendental equation.[1] Sketch a graphical solution.

3. Now assume m/M is small and find the leading order change in the frequency of the fundamental mode.

4. Find the beat frequency between the loaded and unloaded fundamental modes for small values of m/M. If the beat frequency is 3.3Hz, and the unloaded fundamental frequency is 330Hz (the open E string), what must be the ratio m/M?

[1] A "transcendental equation" is one involving trigonometric or exponential functions. Such equations cannot be solved algebraically. It is therefore often useful to sketch what is called a graphical solution, which is represented as the intersection point of two curves, if possible usually a curve and a straight line for easy inspection. A sketch like this lets you see qualitatively how the solution behaves as parameters in the equation are changed. In this problem, the relevant parameter is the ratio of the small mass to the string mass.