

*Partial credit will be given for correct work shown. Also, if you miss an earlier part of a problem do not give up on later parts of the problem, even if they require the result of the earlier part. You can get partial credit by just solving the later part in a more general way.*

*This exam is designed to be very straightforward and simple, and most of the problems can be done in very little time. Those that almost certainly require five minutes or under are tagged by [ $\leq 5'$ ]. If you are not getting one of these within five minutes, I suggest you stop and come back to it later, and/or try to think about it in a different (simpler) way. Some of the others may not take you more than five minutes either.*

1. Consider a mass  $m = 2$  moving in a potential  $V(x) = x^4 - 2x^2$ , where all quantities are given in SI units. [2+3+5=10 points]
  - (a) Sketch a graph of  $V(x)$ , clearly showing sign, zeros, stationary points, and behavior at large  $|x|$ .
  - (b) What are the point(s) of stable equilibrium?
  - (c) What is the period of small oscillations about one of these equilibrium points?
2. Let  $z = e^{i\pi/3} - 2i$ . [ $\leq 5'$ ] [3+5+2=10 points]
  - (a) Represent  $z$  by a vector addition diagram in the complex plane.
  - (b) What are the magnitude and phase of  $z$ ?
  - (c) What is  $1/z$  in polar form?
3. An object of mass 2 kg hangs from a spring of negligible mass. the spring is extended by 2.5 cm when the object is attached. The top end of the spring is moved up and down in simple harmonic motion with an amplitude of  $\Delta h = 1$  mm, providing a driving force due to the extra stretching of the spring. The Q of the system is 15. [3+4+4+4=15 points]
  - (a) What is the resonant angular frequency for this system?
  - (b) Show that if the driving frequency is at resonance, the amplitude of the forced oscillation is  $Q\Delta h = 15$  mm.
  - (c) What is the phase of the oscillating position of the mass relative to that of the driving force at resonance?
  - (d) Approximately long does it take for the transient to die away to 1/100 of its initial energy after the driving force is turned on?
4. A guitar string of length 60 cm is plucked while touching the string at a point one third of the way from one end. The resulting vibration has a frequency of 600 Hz, an amplitude of 1 mm, and is the lowest frequency mode with a node at that point. [5+5+5=15 points]
  - (a) What is the speed of transverse waves on this string?

- (b) Write an expression for the transverse position  $y(x, t)$  in this mode with  $x$  in cm and  $t$  in sec.
- (c) If the total energy in this mode is 0.01 J, what are the tension and linear mass density of the string?

(Possibly useful fact: Remember that  $\int_0^{n\pi} d\theta \cos^2 \theta = n\pi/2$  for any integer  $n$ .)

5. Surface capillary waves have a dispersion relation of the form  $\omega^2 = \beta k^3$ . (a) What are the *dimensions* (not units) of  $\beta$ ? (b) If capillary waves with a wavelength of 5 mm have a phase velocity of 5 cm/s, what is the group velocity of a wave packet consisting of wavelengths centered on 5 mm? [  $\leq 5'$  ] [5+5 = 10 points]
6. (a) Write the Maxwell equation in differential form involving the displacement current term, and identify the displacement current itself. (b) Explain what is the inconsistency encountered if the displacement current term is omitted. [  $\leq 5'$  ] [2+3=5 points]
7. A WMUC radio transmitting antenna operates at a (very low) power of 10 W. (a) If you listen to the station at a distance of 1 km from the antenna, what is the direction and average magnitude of the Poynting vector at your location? (b) What is the rms electric field in the waves where you are listening, in volts per meter? [  $\leq 5'$  ] [5+5=10 points]
8. Due to the expansion of the universe, the light from the most distant objects we can see is redshifted in wavelength by an amount  $(\lambda' - \lambda)/\lambda = 6$ , where  $\lambda$  is the wavelength in the rest frame and  $\lambda'$  is the wavelength we observe. If this is interpreted as being due just to relative motion, what would be the recessional velocity of the object relative to us? Give your answer as a fraction of the speed of light  $c$ . [  $\leq 5'$  ] [5 points]
9. Two polarizing filters are crossed so that no light gets through. If a third filter is inserted between the two at an angle of 30 degrees, what is the intensity  $I$  of the transmitted light in terms of the incident unpolarized light of intensity  $I_0$ ? [  $\leq 5'$  ] [5 points]
10. In costume jewelry, rhinestones (made of glass with  $n = 1.5$ ) are often coated with silicon monoxide ( $n = 2.0$ ) to make them more reflective. What is the minimum thickness for the coating to achieve strong reflection for 560 nm light, incident normally? [5 points]
11. Two vertical dipole radio antennas are separated by a distance  $2\lambda$  in the north-south direction, where  $\lambda$  is the wavelength being broadcast. The same signal is fed in phase to both antennae. Suppose the intensity of the radiation from each antenna alone is  $I_0$  at some large distance  $d$ . When both antennae broadcast together and are viewed from the distance  $d$ , (a) How many interference maxima are there around a complete circle encompassing the the two antennae, and at what angles are they located? (b) What is the intensity of each of these maxima? (c) What is the intensity in the SW direction (45 degrees south of west)? [5 + 2 + 3 = 10 points]

Possibly useful numbers:

$$c = 3 \times 10^8 \text{ m/s}, \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2, \quad \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2.$$