

Partial credit will be given for correct work shown (except on the multiple choice questions). Also, if you miss an earlier part of a problem do not give up on later parts of the problem, even if they require the result of the earlier part. You can get partial credit by just solving the later part in a more general way.

This exam is designed to be very straightforward and simple, and most of the problems can be done in very little time. Those that almost certainly require three minutes or under are tagged by [$\leq 3'$]. If you are not getting one of these within three minutes, I suggest you stop and come back to it later, and/or try to think about it in a different (simpler) way. Some of the others may not take you more than three minutes either.

1. Consider a mass m moving radially in a radial potential $V(r) = ar + b/r^2$, where a and b are both positive. [2+3+5=10 points]
 - (a) Sketch a graph of $V(r)$.
 - (b) What is the equilibrium radius r_0 ?
 - (c) What is the period of small oscillations about equilibrium?
2. A damped harmonic oscillator involves a block of mass m , and spring of constant k , and a damping force $-bv$. The amplitude falls to $e^{-1/4}$ (≈ 0.78) of its initial value after four cycles. [4+2+5=11 points]
 - (a) What is b ? (You can use the small damping approximation to simplify the algebra.)
 - (b) What *dimensionless* quantity should be small in order for the small damping approximation to be accurate? Show that your solution to (a) satisfies this condition.
 - (c) Suppose the damped oscillator is now driven by a force $F(t) = F_0 \cos \omega_0 t$ at precisely the natural frequency $\omega_0 = \sqrt{k/m}$ of the undamped oscillator. After the transients have died away, what is the amplitude and frequency of the resulting oscillation, and what is its phase relative to that of the driving force?
3. An air conditioner connected to an AC line with voltage $\mathcal{E}(t) = \mathcal{E}_0 \cos \omega t$ is equivalent to a resistance R and inductance L in series. (a) What is the magnitude of the impedance of the air conditioner? (b) What is the current in the resistor? (c) What is the average power supplied to the appliance? [3+4+3=10 points]
4. A transverse wave on a string of linear mass density μ is described by the displacement $y(x, t) = a \sin(bx + ct + d)$. In terms of a, b, c, d find the (a) amplitude, (b) wavelength, (c) frequency (not angular), (d) velocity (magnitude and direction), (e) maximum transverse speed of a particle in the string, (f) tension of the string? [6×2=12 points]
5. An alarm clock emits a sound with sound level 40 dB at a distance of 2 m in all directions. What is the maximum distance at which the alarm can be heard by someone with a hearing threshold of 0 dB? [$\leq 3'$] [5 points]

6. Surface waves on deep water have a dispersion relation $\omega^2 = gk$, where g is the acceleration of gravity. For waves with a period of 1 s, what is the (a) phase velocity, (b) group velocity, (c) wavelength? [2+2+2 = 6 points]
7. A conducting sphere of radius R is being charged by a current I flowing through a thin wire connected to the sphere at one point. What is the displacement current through the surface of the sphere? [$\leq 3'$] [5 points]
8. A WMUC radio transmitting antenna is a vertical electric dipole. In your dorm room you have a magnetic dipole receiving antenna in the shape of a circular loop, which works by Faraday's law: a changing magnetic flux through the loop generates an emf in the loop. If the vertical direction is \hat{z} , and the direction to the transmitter is \hat{x} , your receiving antenna will pick up a signal if it is in (choose one): (a) the x - y plane; (b) the y - z plane; (c) the x - z plane; (d) {(a) & (b)}; (e) {(b) & (c)}; (f) {(a) & (c)}; (g) all of the above; (h) none of the above. [$\leq 3'$] [5 points]
9. (a) What is a typical wavelength of visible light?
 (b) What type of electromagnetic radiation might have a wavelength of 10 cm?
 (c) What type of electromagnetic radiation might have a wavelength of 1 A?
 [$\leq 3'$] [2+2+2 = 6 points]
10. The mass of a giant black hole in the center of a galaxy can be inferred from the orbital radius and orbital velocity of gas orbiting the hole. The velocity of the gas is determined by the Doppler shift of spectral lines. Suppose we are looking edge on at a disk of such gas, and the difference in wavelength between the lines from the receding and approaching sides of the disk at a given radius is $\Delta\lambda$ for a line of natural wavelength λ_0 . What is the speed v of the gas at this radius? Assume $v/c \ll 1$ and neglect terms of order v^2/c^2 . [5 points]
11. The critical angle for total internal reflection in a material immersed in air is measured and found to be θ_c . What is the speed of light in that material? [$\leq 3'$] [5 points]
12. If you face south at sunset, with the sun directly to your right, how will the the sky light in the direction you are facing be polarized (choose one): (a) horizontally; (b) vertically; (c) diagonally; (d) left circular; (e) right circular; (f) unpolarized. [$\leq 3'$] [5 points]
13. Two vertical dipole radio antennas are separated by a distance $\lambda/4$ in the north-south direction, where λ is the wavelength being broadcast. The same signal is fed to both antennae, but with a phase lag of $\pi/2$ for the northernmost one relative to the other one. Suppose the intensity of the radiation from each antenna alone is I_0 at some large distance d . When both antennae broadcast together what is the intensity, at the distance d , towards (a) the north, (b) the south, (c) the east, (d) the west?
 [10 points]
14. If the central, first, and second interference maxima of a double slit interference pattern are present, but the third interference maximum (not including the central one) is missing, what is the ratio of the slit width to the separation between the slits?
 [$\leq 3'$] [5 points]