Final Exam — Phy171—Spring 2002
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- Answers without reasoning and/or calculations receive no credit.
- Maximize partial credit by NEAT work and CLEAR REASONING.
- Work out formulae before putting in numbers. Check dimensions and limits to spot errors. Include units where applicable.

1. A particle of mass 2 kg makes a displacement \( s = 2 \text{ m}\mathbf{i} - 5 \text{ m}\mathbf{j} \) along a straight line. During the displacement a constant force \( \mathbf{F} = 3 \text{ N}\mathbf{i} + 4 \text{ N}\mathbf{j} + 5 \text{ N}\mathbf{k} \) acts on the particle.
   (a) Find the work done by the force, (b) the component of the force in the direction of the displacement, and (c) the angle between the force and the displacement.

2. If \( \mathbf{F} \) were the only force on the particle in problem 1, the particle would not have made the displacement indicated. If the particle started out at \( \mathbf{r} = 4 \text{ m}\mathbf{k} \) with velocity \( \mathbf{v} = 1 \text{ m/s}\mathbf{j} \), what would its position vector be 2 s later under the action of this force alone?

3. A ski slope 30\(^\circ\) from the horizontal exerts a force of magnitude 400 N on a skier. What is the magnitude of the force the skier exerts on the mountain?

4. A person stands on a scale in an elevator that has a downward acceleration 2 m/s\(^2\). The scale reads 480 N. What is the mass of the person?

5. Deduce the period of circular motion of a small mass hanging on the end of a massless string one meter long and making a fixed angle thirty degrees with the vertical.

6. A 300 g handball moving with a speed of 5 m/s strikes a wall at an angle of 40\(^\circ\) and then bounces off with the same speed at the same angle (reflected across the normal direction). It is in contact with the wall for 2 ms. What is the average force exerted by the ball on the wall?

7. The position vector of a particle of mass 3 kg is given by \( \mathbf{r} = 4t \mathbf{i} + 3t^2 \mathbf{k} \), where \( \mathbf{r} \) is in meters and \( t \) is in seconds. Determine the angular momentum and the torque acting on the particle about the origin as a function of time.

8. A newly discovered comet enters the solar system and makes a pass around the sun. The distance of closest approach is \( b \) and at that point the speed is \( v_0 \). Suppose the comet’s orbit is elliptical. Write down two independent equations involving the speed \( v \) and distance \( r \) from the sun at aphelion (the most distant point) which could be solved simultaneously to determine \( v \) and \( r \) in terms of \( v_0 \) and \( b \). (Hint: Use two different conservation laws.)

9. A 2 kg block sits atop a 4 kg block that sits upon a frictionless table. The coefficient of static friction between the blocks is \( \mu_s = 0.3 \). What is the maximum horizontal force \( F \) that can be applied to the lower block if the upper block is not to slide?
10. The potential energy of a 4 kg object is given by \( U = 3x^2 - x^3 \) where \( x \) is in meters and \( U \) is in joules. (a) At what positions is this object in equilibrium? (b) Sketch a decent plot of \( U \) vs. \( x \). (Do not use your calculator to do this. It may well give you a misleading result.) Label clearly the location of the zeros and stationary points. (c) For each equilibrium point state whether it is stable, unstable, or neutral. (d) What is the maximum speed the particle can have at \( x = 0 \) if its subsequent motion is to remain bounded (i.e. not to run off to \( x = \infty \))? 

11. A particle of mass \( m \) moving at speed \( v_0 \) in the “lab frame” collides with a particle of mass \( 2m \) at rest. (a) What is the velocity (magnitude and direction) of the mass \( m \) in the center of mass frame before the collision? What is the velocity of the mass \( m \) in the center of mass frame after the collision if (b) the collision is perfectly inelastic, or (c) perfectly elastic? (d) What is the final velocity of the mass \( m \) in the lab frame in the elastic case? 

12. A basketball (a hollow spherical shell) of mass \( M = 400 \) g and radius \( R = 12 \) cm rolls without slipping down a road which makes an angle of 30 degrees with the horizontal. What is the magnitude and direction of the friction force that acts on the ball? 

13. A thin bar of length \( L \) and mass \( M \) lies at rest on a frictionless horizontal surface. A small hard sphere of negligible size moving at speed \( v_0 \) perpendicular to the rod collides elastically with the rod at a distance \( d \) from its center. Find \( d \) such that the sphere is at rest after the collision. Does your answer have the right limit when \( m = M \)? 

14. A hollow cube with edge \( a \) is half-filled with water of density \( \rho \). Find the force exerted on a side of the cube by the water. 

15. A scuba diver is 40 m below the surface of a lake, where the temperature is 5 °C. She releases an air bubble with a volume of 15 cm³. The bubble rises to the surface, where the temperature is 25 °C. What is the volume of the bubble right before it breaks the surface? (Hint: Remember that the pressure also changes.)