Please do all problems, and show your work clearly. Credit will not be given for answers with no work shown. Partial credit will be given. Note: in some or all of these problems, you will need the electromagnetic constant k=9x10^{9}Nm^2/C^2=1/(4πε$_0$). Please read the problems first, you don't have to do them in any particular order.

Problem 1 (20 points). A charge of 4nC ("n"≡10^{-9}) is moved from infinity to a point 15cm away from a charge of 8nC.
a) Calculate the potential energy needed to do this.
b) Calculate the voltage at a point that forms an equilateral triangle as in the figure.
c) How much potential energy is needed to move a 7nC charge from infinity to that point (the top of the equilateral triangle as in the figure)?
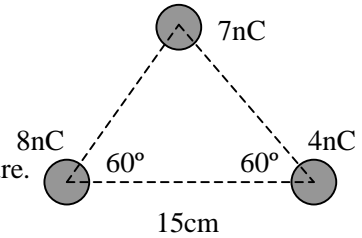
a) The voltage at a point 15cm away from the 8nC charge is given by:
$$V = \frac{kQ}{r} = \frac{9x10^9 \bullet 8x10^{-9}}{0.15} = 480 Volts$$ where we take the reference voltage to be V(∞)=0 The potential energy needed to move a 4nC charge to that position from infinity is given by the product of the charge and the voltage: $U = qV = 4x10^{-9} \bullet 480 = 1.92x10^{-6} J$

b) The point at the top of the equilateral triangle is also 15cm away from both the 8nC and the 4nC charge. The voltage at that point is just the sum of the voltage due to each charge, which is given by $V_1 = \frac{kQ}{r} = \frac{9x10^9 \bullet 8x10^{-9}}{0.15} = 480 Volts$ from the 8nC charge and

$V_2 = \frac{kQ}{r} = \frac{9x10^9 \bullet 4x10^{-9}}{0.15} = 240 Volts$ from the 4nC charge, or $V_{total} = V_1 + V_2 = 720 Volts$

c) The potential energy needed to move a 7nC charge to that point is again the product $U = qV = 7x10^{-9} \bullet 720 = 5.04x10^{-6} J$

Problem 2 (20 points). On a clear sunny day, it's a fact that close to the surface of the earth, a constant vertical electric field of about **130 N/C** points down over the flat ground. This field is actually due to the charging of the earth from lightening and other natural phenomenon. Calculate
a) The surface charge density on the ground for these conditions,
b) The potential difference between the ground and a point **1m** above the ground.

a) the constant electric field is generated by the surface charge density on the ground, and is given by E=σ/2ε$_0$=130N/C. solve for
σ =2 ε$_0$E=2•8.85x10^{-12}C^2/Nm^2•130N/C=2.3x10^{-9}C/m^2

b) the relationship between a constant electric field and the potential difference across a distance Δx is given by ΔV=EΔx. this gives ΔV=130V/m•1m=130V. note that voltage decreases in the direction of the electric field (remember the positive point charge has an electric field which points away from the charge, and a potential that decreases from infinity at r=0 to zero at r=infinity). Since the field points down, then the voltage decreases from 130V at 1m to 0 volts at the ground.

Problem 3 (20 points). In the figure, 2 charges of 9µC and 4µC are placed a distance 4m apart.
a) Calculate the magnitude and direction of the force on a positive 5µC
charge at the midpoint between the two charges.
b) Is there a position somewhere between the two charges where the force
on the 5µC charge is 0? If so, calculate it.

9µC   4µC

4m
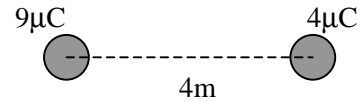
a) the force on one charge due to another is given by $\vec{F}_{12} = \dfrac{kQ_1Q_2}{r^2}\hat{r}$. midway between the 2

charges, the 5µC charge will be 2m away from both. the force from the 4µC charge will be towards the left, and the force from the 9µC charge will towards the right, but since the 5µC charge is the same distance from each, the force due to the 9µC charge will dominate and so the net force will be to the right. the value of the force will be given by the difference between the 2 forces since these forces point in opposite directions, and is

$$F = \frac{9x10^9\, N\,m^2/C^2 \bullet 5x10^{-6}C}{2m^2} \bullet \left(9x10^{-6}C - 4x10^{-6}C\right) = 0.056N$$

b) let the variable x measure the distance from the 4µC charge. then the net force on the 5µC

charge will be given by $F = \dfrac{k \bullet 5\,\text{\textit{n}}C \bullet 9\,\text{\textit{n}}C}{\left(4-x\right)^2} - \dfrac{k \bullet 5\,\text{\textit{n}}C \bullet 4\,\text{\textit{n}}C}{x^2} = 0$. solving for x

gives $\dfrac{9}{\left(4-x\right)^2} - \dfrac{4}{x^2} = 0$ or $\dfrac{4-x}{3} = \pm\dfrac{x}{2}$ after inverting, which gives x=(-8m, 8/5m). since we are

only interested in the solution between, the answer is x=8/5m.

Problem 4 (20 points). The figure shows a hollow conducting shell.
A total charge of 10nC is dumped onto the conductor surface.
a) For an arbitrary circular surface (dotted circle) centered at the object, calculate
the total flux of electric field lines through the surface.
b) If the potential on the face of the surface facing you is measured to be 10V,
calculate the potential on the opposite face, and the potential on the curved side.
c) For the same surface potential as in part b), calculate the potential inside the shell.
d) For the same surface potential as in part b), calculate the electric field inside the shell.
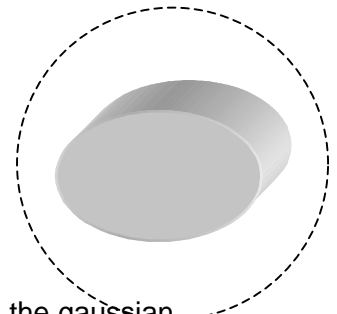
a) the electric flux is defined as the integral of all the electric field lines through the gaussian
surface, and by gauss' law:

$$\Phi_E \equiv \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{e_0} = 4\textbf{\textit{p}}kQ_{inside} = 4\textbf{\textit{p}} \bullet 9x10^9\, Nm^2/C^2 \bullet 10x10^{-9}C = 1130.97 Weber$$

b) since it is a conductor, and since a conductor maintains an equipotential surface, the potential
is the same on all faces of the figure: 10V
c) since it is a conductor, the potential inside is constant and equal to the potential at the
surface, 10V
d) also since it is a conductor, there is no net electric field inside. you can also see this by
gauss' law.

Problem 5 (20 points). Find the equivalent capacitance of the circuit in the figure below.

the 4µF capacitor on the right is in series with the 3µF capacitor on the bottom, and they form an

equivalent capacitance given by $\dfrac{1}{C'_{eq}} = \dfrac{1}{3} + \dfrac{1}{4} = \dfrac{7}{12}$ which gives $C'_{eq} = 12/7\, \textit{mF}$. this capacitor is now

in parallel with the 2µF capacitor, which gives an equivalent capacitance
$C''_{eq} = 12/7 + 2 = 26/7\, \textit{mF}$ and that capacitor is in series with the 1µF capacitor, which gives an

overall equivalent capacitance of $\dfrac{1}{C_{eq}} = \dfrac{1}{1} + \dfrac{7}{26} = \dfrac{33}{26}$ or $C_{eq} = 26/33\, \textit{mF} = 0.79\, \textit{mF}$