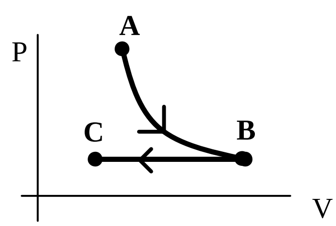
Physics 262, Exam 2, April 4, 2002, Dr. Baden

Please do all problems, and show your work clearly. Credit will not be given for answers with no work shown. Partial credit will be given. Note: in some or all of these problems, you will need the gas constant R=8.3 J/moleK. If you convert this to have units of Liter-atm/K you might find some of the calculations easier. So, 1atm = $1.01\times10^5 N/m^2$ and 1Liter = $10^{-3} m^3$ and 1 Liter-atm = 101J, this gives the value R=0.083Liter-atm/moleK.

Problem 1 (30 points). **One mole** of an ideal gas is taken through the sequence of changes shown on the *p-V* diagram in the figure: the state starts at point A, expands isothermally to point B, and is compressed at constant pressure to point C. **P_A=5.00atm**, **V_A=8.00Liters**, **V_B=40.0Liters**.

a) What is the temperature T_A?
b) What is the pressure P_B?
c) If $T_C = T_A/8$ what is V_C?



For this problem, use the gas constant in units of L-atm: R=0.083 L-atm/moleK.

a) PV=nRT therefore $T_A=P_AV_A/nR$=5.0atm•8.0L/(1mole•0.083L-atm/moleK)=481.9K.

b) Again, we use the ideal gas law. Since the leg A→B is an isothermal expansion, the temperature at point B is also 481.9K. So we have $P_BV_B=nRT_B$ or $P_B=nRT_B/V_B$=1mole•0.083L-atm/moleK•481.9K/40L=1.0atm

c) $T_C=T_A/8$=481.9k/8=60.2K. $P_CV_C=nRT_C$ and $P_C=P_B$=1.0atm so V_C=1mole•0.083L-atm/moleK•60.2K/1.0atm=5.0L

Problem 2 (30 points). **100grams** of ice at **0°C** is brought in thermal contact with **1kg** of Aluminum at **80°C**. The specific heat of ice is **0.5cal/gm°C**, for water is **1.0cal/gm°C**, and for Aluminum is **0.215cal/gm°C**. The heat of vaporization of water is **540 cal/gm** and Aluminum is **2723 cal/gm**. The heat of fusion for water is **80 cal/gm** and for Aluminum is **95 cal/gm**. Calculate the final temperature of the H_20/Aluminum system.

You first have to melt all the ice at a constant temperature of 0°C. Use the heat of fusion of ice to get $Q_{melt}=m_{ice}\bullet L_f$=100g•80cal/gm=8000cal. Then, you have to calculate the temperature drop of the Aluminum in releasing this much heat to the ice, using Q_{drop}= Q_{melt}=8000cal=$m_{Al}\bullet c_{Al}\bullet\Delta T_{Al}$=1000g•0.215cal/gm°C•$\Delta T_{Al}$ and solve for ΔT_{Al}. You then get ΔT_{Al}=37.2°C. Therefore the temperature of the Al dropped by 37.2°C to 80-37.2=42.8°C. Now you have 100g of water at 0°C and 1000g of Al at 42.8°C so you find the final temperature. The heat capacity of the Al is given by $C_{Al}=m_{Al}c_{Al}$=1000g•0.215cal/gm°C=215cal/°C. The heat capacity of the water is given by $C_w=m_wc_w$=100g•1.0cal/gm°C=100cal/°C.

You can use the formula that we derived in class but you have to use K, not °C:

$T_f = \frac{C_W T_W + C_{Al} T_{Al}}{C_W + C_{Al}}$ =[100(273+0) + 215(273+42.8)]/(100+215)=301K=29.2°C.

Alternatively, you can just set the heat lost by the hotter aluminum equal to the heat gained by the colder water: $Q_{Al,lost}=C_{AL}(T_{Al}-T_f)=Q_{w,gain}=C_w(T_f-T_w)$ and solve for T_f.

Problem 3 (40 points). One mole of an monatomic ideal gas is taken around the cycle shown in the figure. In legs A→B and C→D the volume is held fixed at $\mathbf{V_1=1.0Liters}$ and $\mathbf{V_2=3.0Liters}$ respectively. In legs B→C and D→A the pressure is held fixed at $\mathbf{P_2=4.0atm}$ and $\mathbf{P_1=2.0atm}$ respectively.

a) For each leg, calculate
   1. The work done by or on the gas. Specify whether the work is done on the gas, or the gas does work.
   2. The heat absorbed or expelled. Specify whether the heat is absorbed, or expelled, by the gas.
   3. The change in the internal energy of the gas. Specify whether the internal energy went up, or down.

b) Calculate the total work done by the system.

c) Is this an engine or a refrigerator?

d) If it's an engine, calculate the efficiency of the system. If refrigerator, calculate the coefficient of performance.

First we'll do each leg separately:

A→B. This is a raising of the pressure at constant volume. So the work done is zero: **W=0**. The change in pressure causes a change in temperature, so $\Delta PV=nR\Delta T$. You can solve for $\Delta T=\Delta P\bullet V/nR$. For monatomic ideal gasses, the internal energy change as a function of the temperature change is given by $\Delta U=(3/2)nR\Delta T$. So we have $D\mathbf{U}=(3/2)nR\Delta T=(3/2)nR\bullet\Delta P\bullet V/nR=(3/2)\Delta P\bullet V=1.5\bullet(4atm-2atm)\bullet 1.0Liter=$**3.0Liter-atm**. The internal energy went up since the pressure went up causing an increase in the temperature. By the first law, $Q_{in}=W_{by}+\Delta U$ so $\mathbf{Q_{in}=3.0Liter\text{-}atm}$. Since Q_{in} is positive, heat is absorbed by the gas.

B→C. This is a constant pressure process. The work done will be given by $\mathbf{W}=P\Delta V=4.0atm\bullet(3.0-1.0Liter)=$**8.0Liter-atm**. Work is positive, volume increases, the gas is expanding and doing work. The change in the internal energy is again given by $\Delta U=(3/2)nR\Delta T$. Here ΔT is calculated from the idea gas law: at constant pressure, $P\Delta V=nR\Delta T$ so we have $D\mathbf{U=}(3/2)nR\Delta T=(3/2)nR\bullet P\Delta V/nR=(3/2)P\Delta V=1.5\bullet 4.0atm\bullet(3.0-1.0Liter)=$**12.0Liter-atm**. The internal energy is going up because the volume is increasing at constant pressure, and this can only happen by increasing the temperature of the gas. By the first law, $Q_{in}=W_{by}+\Delta U$ so $\mathbf{Q_{in}=}8.0+12.0Liter\text{-}atm=$**20Liter-atm**. Heat is added to the system, and this heat causes the temperature to increase, the volume to expand, and work is done by the gas.

C→D. The pressure drops. No work is done because the volume is constant: **W=0**. The drop in pressure is causes a drop in temperature, and heat is released by the gas. Just as in the first leg, $D\mathbf{U}=(3/2)nR\Delta T=(3/2)nR\bullet\Delta P\bullet V/nR=(3/2)\Delta P\bullet V=1.5\bullet(2.0-4.0atm)\bullet 3Liter=$**–9Liter-atm**. The internal energy decreases. Since W=0, the heat lost comes from the lost internal energy, and $Q_{in}=\Delta U=-9Liter\text{-}atm$, so $\mathbf{Q_{out}=9Liter\text{-}atm}$.

D→A. Just like the leg from B to C, only here the volume decreases. Work is therefore done **on** the gas and is given by $\mathbf{W_{by}=}P\Delta V$=2.0atm•(1.0-3.0Liter)=**–4.0Liter-atm**. Since work is negative, this means work is done on the gas. As the volume drops at constant pressure, the temperature drops, decreasing the internal energy of the gas, which is given by D**U=**(3/2)nRΔT=(3/2)nR•PΔV/nR=(3/2)PΔV=1.5•2.0atm•(1.0-3.0Liter)= **–6.0Liter-atm**. Heat is drawn out of the gas as the temperature drops. By the first law $Q_{in}=W_{by}+\Delta U$ but here both W_{by} and ΔU are negative, which means Q_{in} is negative, or Q_{out} is positive. So we have $\mathbf{Q_{out}=}W_{on}+\Delta U_{drop}$=4.0+6.0Liter-atm = **10Liter-atm**.

The total work done by the system will be given by the work done **by** the gas minus the work done **on** the gas, or 8Liter-atm–4Liter-atm = 4Liter-atm. You can also calculate this easily by taking the area inside the rectangle, given by $\mathbf{W_{total}=}\Delta P\Delta V$=(4atm–2atm)(3Liter–1Liter)=**4Liter-atm**.

Since the total work is positive, this is an engine.

The efficiency is given by the ratio of the W_{total} to the Q_{in}. Heat is absorbed during the legs A→B and B→C, where the state variables are increasing: Q_{in}=3.0Liter-atm + 20Liter-atm=23Liter-atm. The efficiency is therefore e**=**4Liter-atm/23Liter-atm=**0.174=17.4%**