Physics 262, Exam 1, March 5, 2002, Dr. Baden

Please do all problems, and show your work clearly. Credit will not be given for answers with no work shown. Partial credit will be given.

Problem 1 (20 points). A mass of **1.50kg** stretches a vertical spring **0.315m**. The spring is then stretched an additional **0.130m** and released.

a) What is the frequency of the oscillation?
b) At what time $\mathbf{t_1}$ after release does the mass reach the equilibrium position?
c) What is the maximum velocity of the mass?
d) Calculate the maximum extension (amplitude)?
e) What is the total kinetic and potential energy at **t=0** sec?
f) What is the total energy at $\mathbf{t=t_1}$ sec?

The spring constant k tells how many newtons of force produce how much stretching (F=kx). Here we have gravity stretching the spring by .315m, and the force of gravity is F=mg=1.5kg x 9.8m/s^2=14.7N. So the spring constant is k=mg/x=14.7N/0.315m=46.7N/m. The angular frequency ω is given by $w = \sqrt{k/m} = \sqrt{46.7/1.5kg}$ = 5.58rad/sec. Since $\omega=2\pi f$ we can solve for the frequency f= $\omega/2\pi$=0.89Hz as the answer for part a). For part b), the period of the oscillation is given by T=1/f=1.13sec. The mass is released at the point of greatest extension, so it will take 1.13sec for it to go to the other extreme and back. It will get to the other extreme in half the period, and it will pass thru the equilibrium position in half of that time. So it will take T/4=.28sec to get back to the equilibrium position. For part c), use the fact that the amplitude is 0.13m, the maximum velocity is always given by $v_{max}=\omega A$ = 5.58rad/sec x 0.13m = 0.73m/sec. For part d), the maximum extension IS the amplitude, 0.13m. For part e), the total kinetic energy at t=0 is zero because this is the point of maximum extension, it's not moving initially, and after 1 or more periods it will be back at this point and turning around. If it's turning around, it can't be instantaneously moving. The total potential energy is given by PE=(1/2)kA^2=.5 x 46.7N/m x (0.13m)^2 =0.395Joules. For part f), since energy is always constant, the total energy is just the initial total energy, which was all PE, so it's the same 0.395Joules. If you wanted to, you could use the fact that the maximum velocity is at the equilibrium position, there's no PE at that point so you would get Etot=(1/2)mv_{max}^2 =0.5 x 1.5kg x (.73m/s)^2 =0.395Joules.

Problem 2 (20 points). At 0.5m away, a normal conversation will register approximately 65dB on a dB-meter. Assume that the power is radiating outwards from a person's mouth uniformly over a hemisphere. Calculate

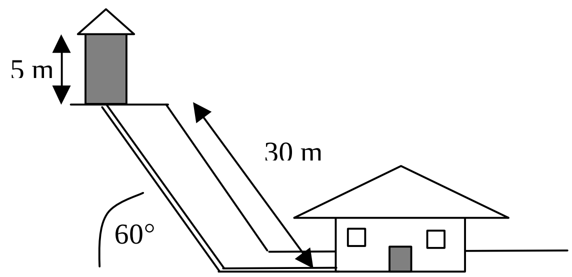
a) The power output of the speaker, in watts.
b) How many people would be required in order to produce a total sound output of 100W of ordinary conversation?

Part a) If the dB meter reads 65dB, then the intensity at a distance 0.5m away can be calculated using the formula $dB = 10\log\left(I/10^{-12}\right)$ so this means $I = 10^{-12} \times 10^{\log(dB/10)}$=3.16x10^{-6}W/m^2. If the power was radiating out in a full sphere, then the intensity would be given by I=P/A where A is the area of a sphere, given by A=4πr^2. However, since the power is radiating out in a HALF sphere, the area would be $A=2\pi r^2$ so the power radiated by the source would be given by $P=IxA=3.16x10^{-6}W/m^2$ x 2π x $(0.5m)^2=4.97x10^{-6}W$. For part b), if 1 person produces $4.97x10^{-6}W$, then it would take N people to produce 100W, and so $Nx4.97x10^{-6}W=100W$ which gives around 32million people!

Problem 3 (20 points). A house at the bottom of a hill gets its water from a cylindrical water tower. The tank is always full of water, is **5m** deep (height) with a diameter of **2m**, and is connected to the house by a **5cm** diameter pipe that is **30m** long at an angle of **60°** from the horizontal. The first floor is located **3.1m** above the main floor, and has a bathroom that has a faucet in the sink. The faucet is **1m** above the floor, and the faucet has a **1cm** diameter.

a) Calculate the water pressure at the 1st floor bathroom faucet.

b) If water comes out of the faucet at **1.2kg/sec**, how long will it take to empty the water tower?



The bottom of the house is a height "h" below the bottom of the tank, where $h=30\sin(60°)=25.9m$. The tank is 5m in height, so first floor is 30.9m below the top of the tank. The pressure at that depth is given by $P=P_0+\rho gh=1.01x10^5Pa$ + $(1000kg/m^3)x9.8m/s^2x30.9m=4.04x10^5Pa$. The pressure a distance 3.1m up would be reduced by an amount ρgh (h=3.1m) and the pressure at the faucet would be reduced by another ρgh (h=1m), so the pressure is then reduced to $4.04x10^5$ - $(1000kg/m^3)x(9.8m/s^2)x4.1m=3.6404x10^5Pa$. This would be the pressure at the faucet that forces the water out. For part b), if the water comes out at 1.2kg/sec, then the volume/sec rate would be given by $V/sec=(1.2kg/sec)/(kg/volume)=(1.2kg/sec)/\rho=1.2x10^{-3}m^3/sec$. The volume of the tower is given by the formula V=Ah where $A=\pi r^2 = \pi(2m\ diameter/2)^2 =3.14m^2$. so the volume is $V=3.14m^2x5m=15.7m^3$. So it will take time T to empty out, given by V=(V/sec)xT so $T=V/(V/sec)=15.7m^3/1.2x10^{-3}m^3/sec=13{,}090sec$ = 3 hours 38 minutes 10 seconds.

Problem 4 (20 points). A rectangular tub made of a thin shell of poured cement has length **L=80cm**, width **W=120cm**, and depth **D=50cm** and mass **M=200kg**. 3 people of mass **80kg** each are standing in the tub. How far below the surface of the water will the bottom of tub reach?

The buoyant force must hold up the mass of the 3 people plus the mass of the tub, which is M=200kg+3*80kg=440kg. This buoyant force must be equal to the mass of the water displaced, $M_{disp}=\rho_w V_{displ} g=1000kg/m^2$ x V_{displ} x $9.8m/s^2$. The volume displaced is given by the area of the tub, $LxW=0.96m^2$, times the distance of the tub below the surface, which is what we are solving for. So we have 440kg= $M_{disp}=1000kg/m^2$ x $9.8m/s^2$ x $0.96m^2$ x D, solving for D gives D=0.046m=4.6cm.

Problem 5 (20 Points). A **2m** rope hangs from the ceiling. The rope has a mass of **50grams**, and there is a **25kg** mass attached to the end of the rope. If you bang on the mass with a hammer, it will send a pulse up the rope, the pulse will be reflected at the rope/ceiling boundary, and travel back to the mass. How long will the round trip take for the pulse? (Ignore the mass of the rope when calculating any Tensions.)

The tension in the rope is dominated by the mass hanging from the rope, which is 25kg. So the tension is T=mg=25kg x 9.8m/s^2 = 245N. The mass /length of the rope is given by $\mu=M_{rope}/L_{rope}$=0.05kg / 2m = .025kg/m. So, the velocity of waves on the rope is given by $V=\sqrt{T/m}=\sqrt{245/0.025}$ =99.0m/s. If the rope is 2m and the waves travel at 99m/sec, then the wave will have to make a round trip of 4m and that will take time T=4m/ (99m/sec) = 0.04sec.