Application of the Wigner Distribution to Non-equilibrium Problems at Surfaces: Relaxation, Growth, and Scaling of Capture Zone

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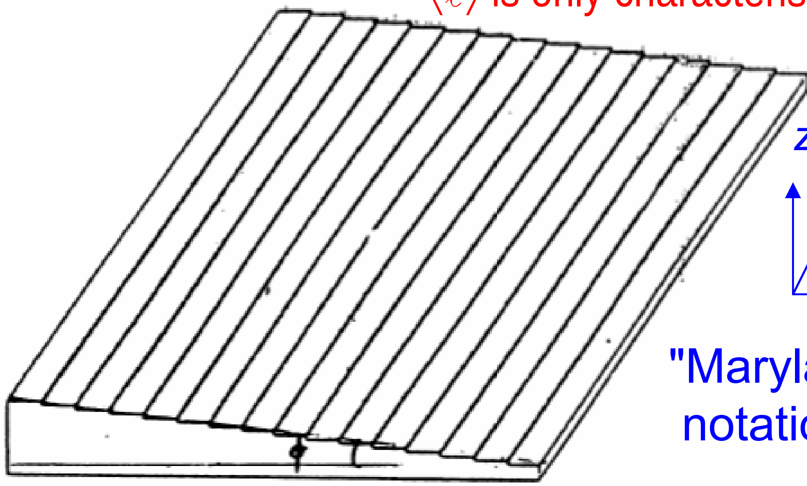
- Summary of equilibrium terrace-width distributions, Wigner surmise (WS) from RMT $P_\beta(s) = a_\beta s^\beta \exp(-b_\beta s^2)$ (β = 1,2,4 for ensembles with orthogonal, unitary, symplectic symmetry) and applications
- Generalization to arbitrary positive $\beta \to \varrho$ (GWS), with no underlying symmetry; applications to surface problems, with ϱ related to step repulsion strength
- Fokker-Planck formulation: study of relaxation to equilibrium & way to get GWS
- Apparent narrowing during growth
- Remarkable progress in characterization of island growth by focusing on capture-zone distribution – GWS with $\varrho = i + 1$ [or $2(i+1)$ in 1D]

Terrace-Width Distribution $P(s)$ for Special Cases

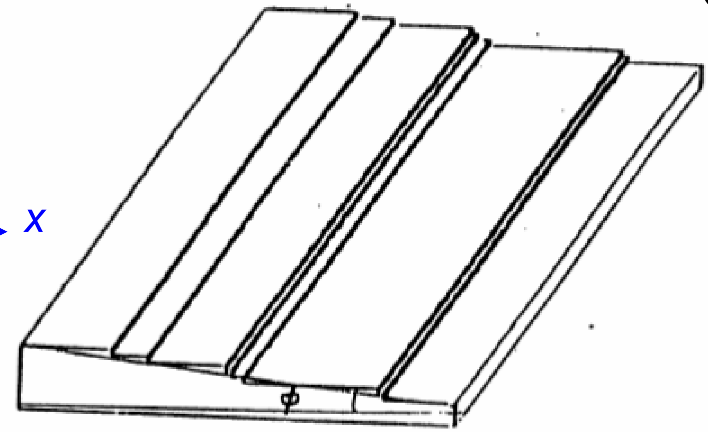
"Perfect Staircase" $\ell = \langle \ell \rangle \equiv 1/\tan \phi$ $s \equiv \ell / \langle \ell \rangle$

$\langle \ell \rangle$ is only characteristic length in x



Straight steps, randomly placed

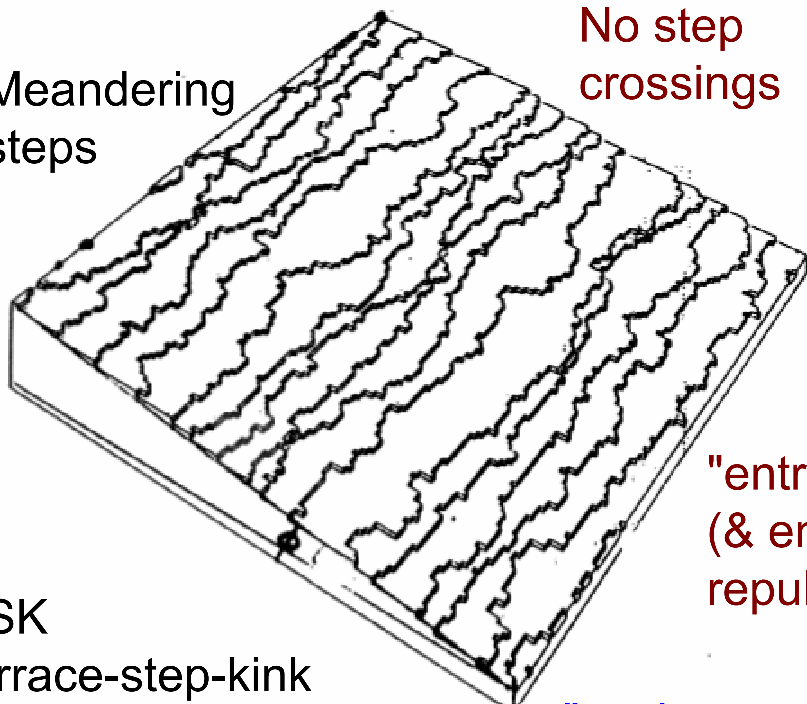
Geometric distribution: $P(s) = e^{-s}$



z, y, x

"Maryland notation"

Meandering steps

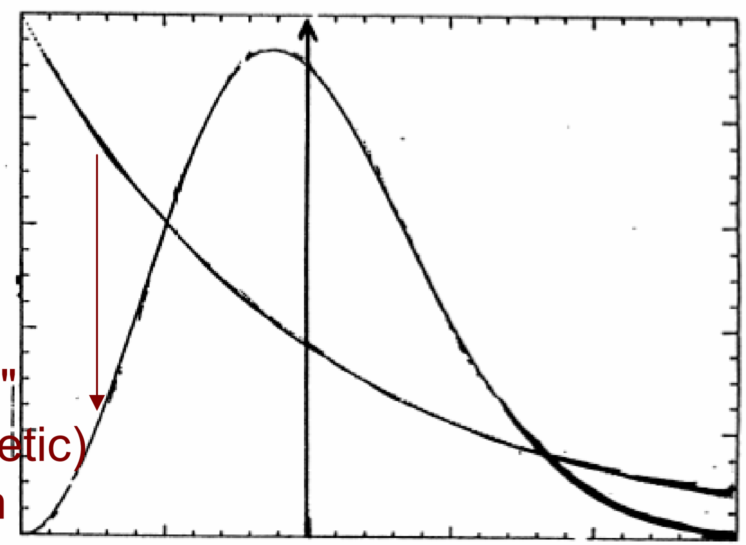


No step crossings

TSK terrace-step-kink

kink energy ε

Scaled TWD: $P(s)$ indep. of $\langle \ell \rangle$

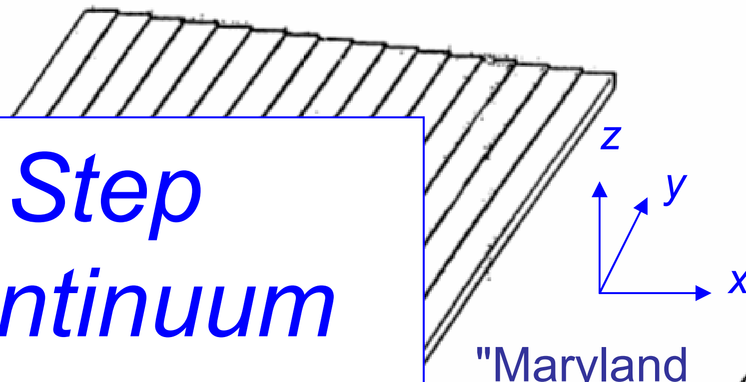


"entropic" (& energetic) repulsion

"static correlation" $\langle x_n(y) - x_{n-1}(y) - \langle \ell \rangle \rangle_{y,n}$

Terrace-Width Distribution $P(s)$ for Special Cases

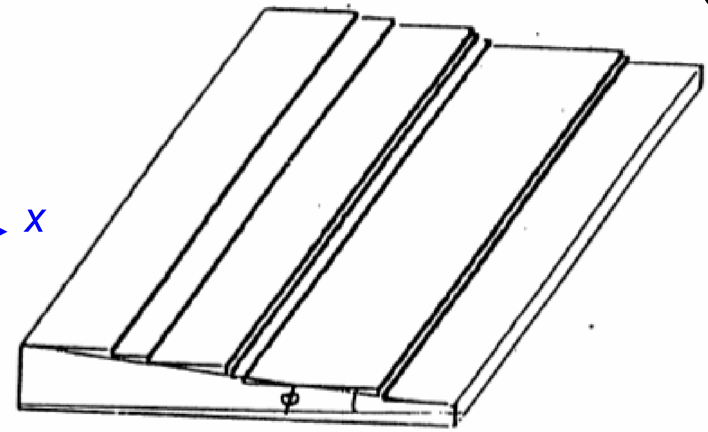
"Perfect Staircase" $\ell = \langle\ell\rangle \equiv 1/\tan\phi$ $s \equiv \ell/\langle\ell\rangle$



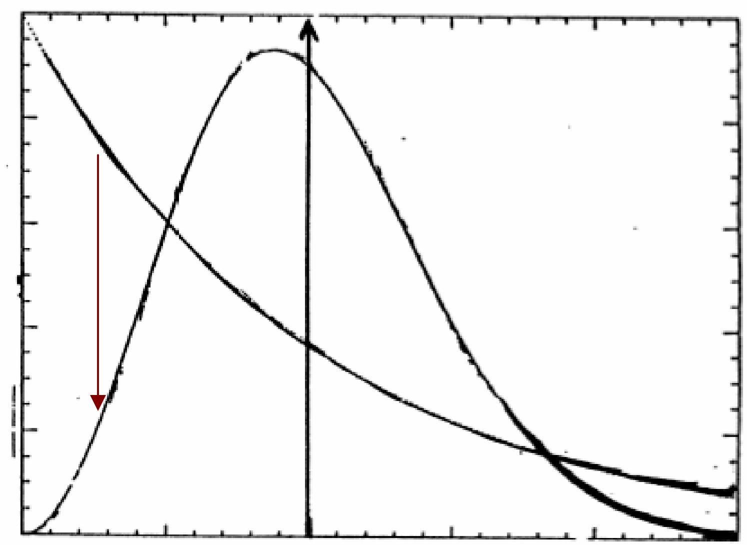
Step Continuum Model

"Maryland notation"

Straight steps, randomly placed
Geometric distribution: $P(s) = e^{-s}$

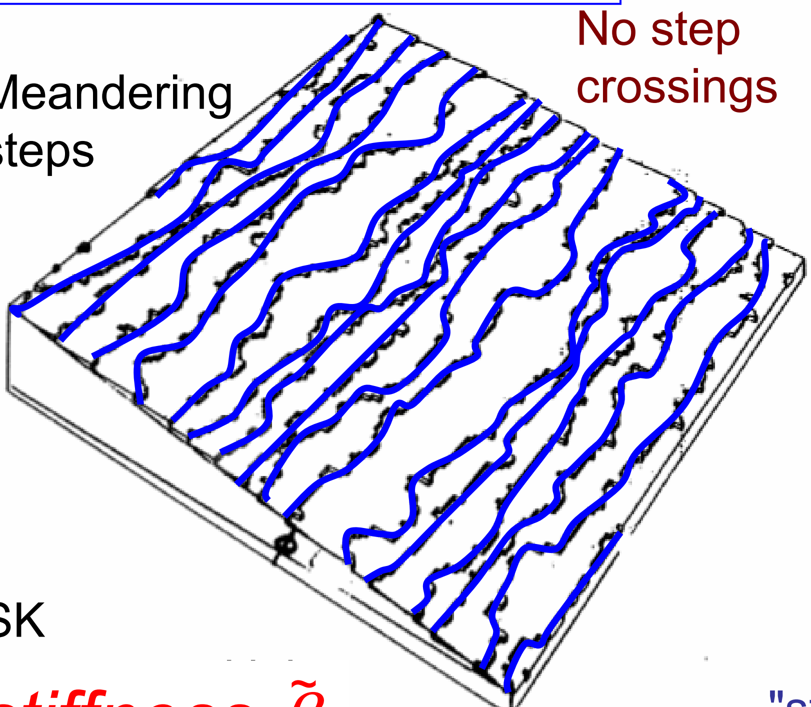


Scaled TWD: $P(s)$ indep. of $\langle\ell\rangle$



Meandering steps

No step crossings



TSK

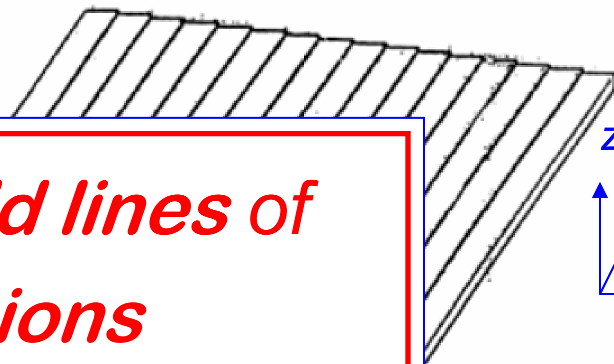
stiffness $\tilde{\beta}$

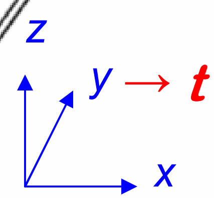
0 1 2 s

"static correlation" $\langle x_n(y) - x_{n-1}(y) - \langle\ell\rangle\rangle_{y,n}^2$

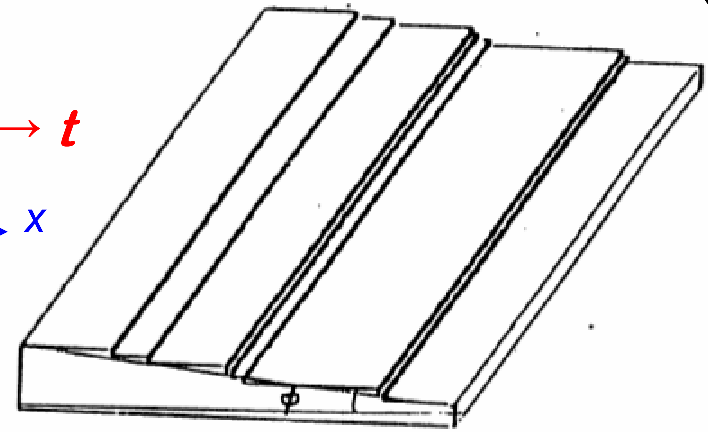
Terrace-Width Distribution $P(s)$ for Special Cases

"Perfect Staircase" $\ell = \langle \ell \rangle \equiv 1/\tan \phi$ $\quad s \equiv \ell / \langle \ell \rangle$





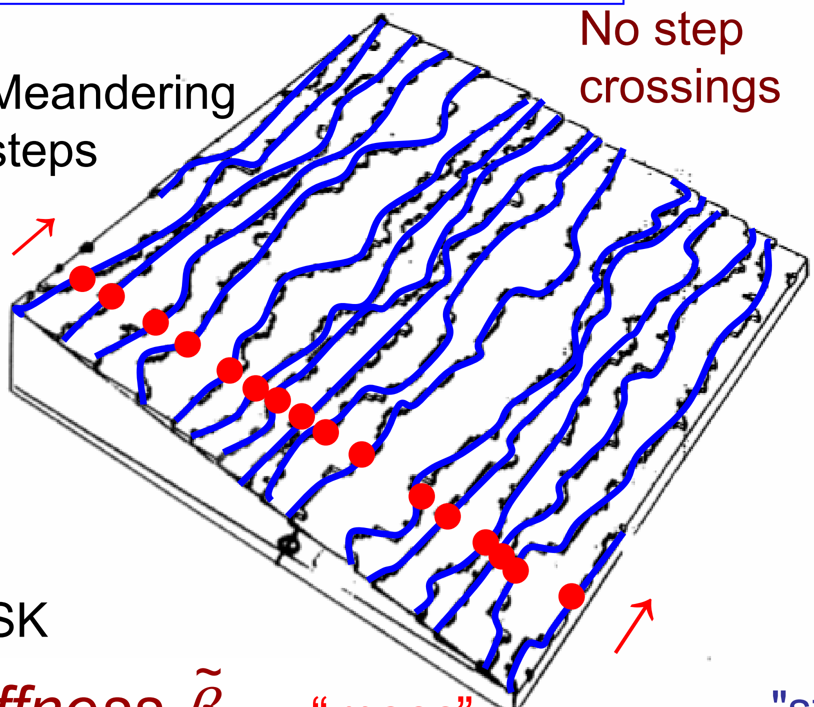
Straight steps, randomly placed
Geometric distribution: $P(s) = e^{-s}$



World lines of fermions evolving in 1D

Meandering steps

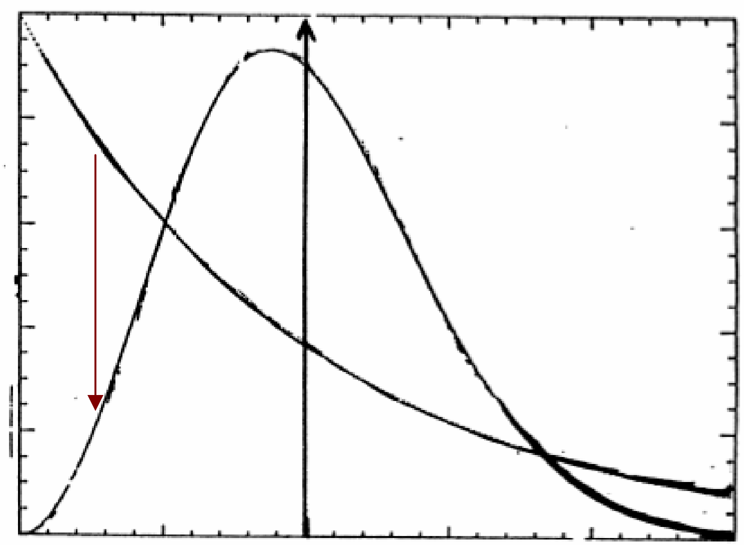
No step crossings



TSK

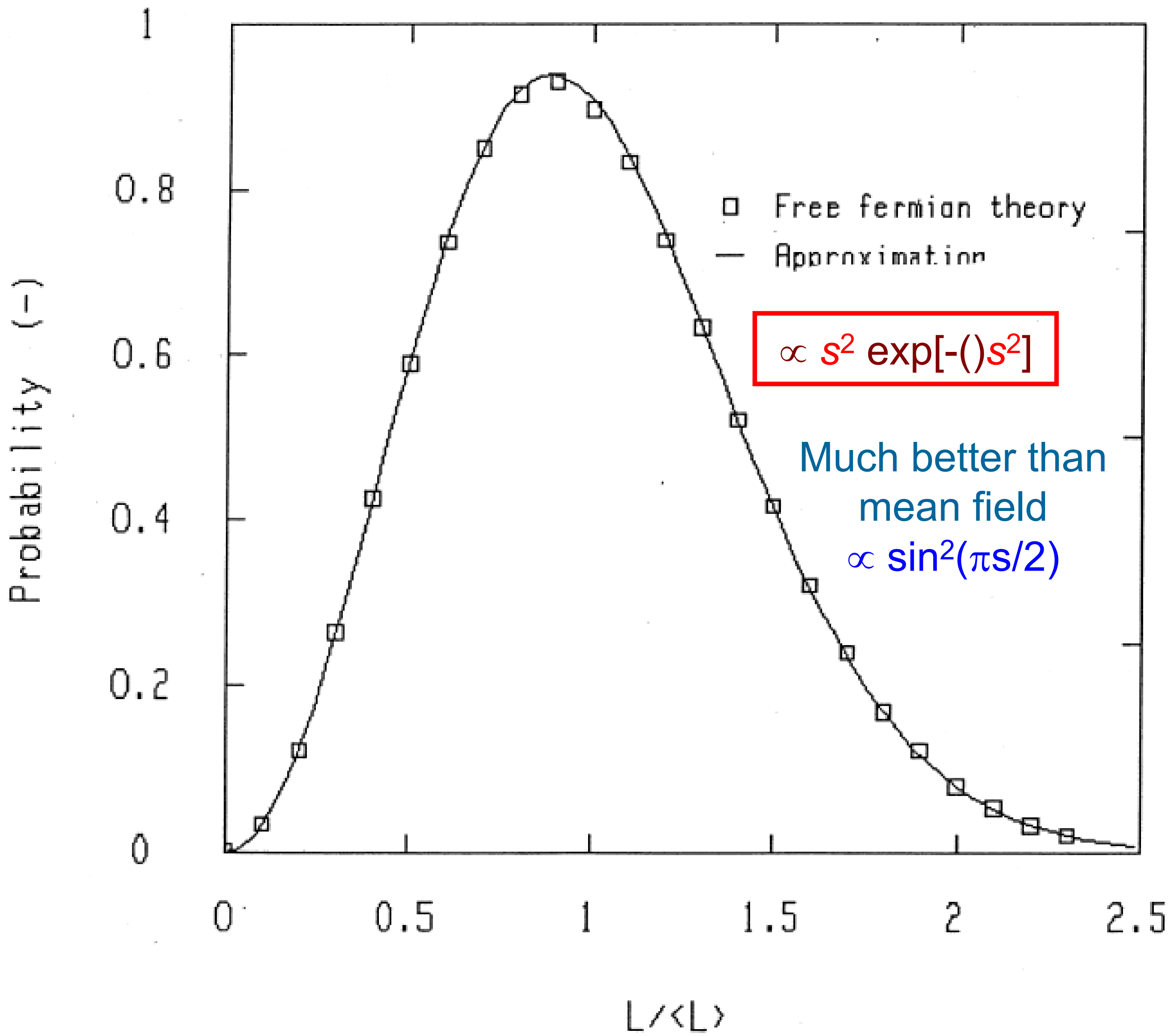
stiffness $\tilde{\beta} \rightarrow$ "mass"

Scaled TWD: $P(s)$ indep. of $\langle \ell \rangle$



"static correlation" $\langle x_n(y) - x_{n-1}(y) - \langle \ell \rangle \rangle_{y,n}$

Deduced & drawn by Harald Ibach (¿while on sabbatical at UM, spring 1997?)

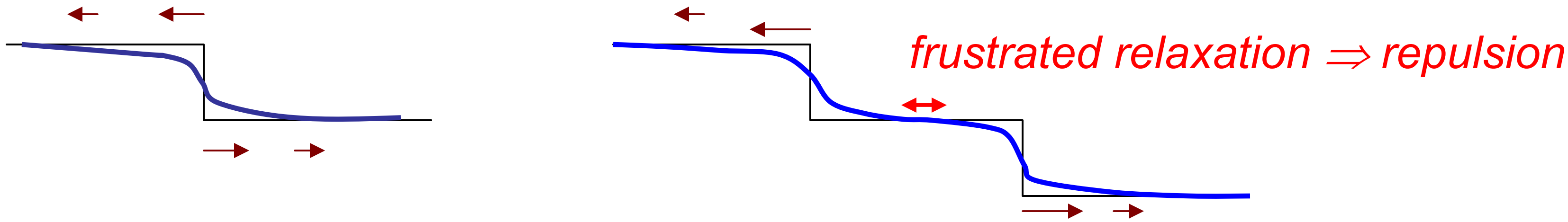


$\propto s^2 \exp[-()s^2]$

Much better than mean field $\propto \sin^2(\pi s/2)$

Origin of elastic (dipolar) step repulsions

- Frustration of relaxation of terrace atoms between steps



- Energy/length: $U(\ell) = A/\ell^2$ (Same y for points on two interacting steps separated by ℓ along x ⇒ "instantaneous")
- Metallic surface states ⇒ added oscillatory term in U: $(B/\ell^2)\cos(2k_F\ell + \phi)$

- Elastic and entropic repulsions $\propto \ell^{-2}$

  ⇒ universality of $\langle\ell\rangle^{-1}P(\ell)$ vs. $s \equiv \ell/\langle\ell\rangle$ so $P(s;\langle\ell\rangle) \rightarrow P(s)$ *scaling*

Physical Ideas Behind Application of Random Matrices

cf. T. Guhr, A. Müller-Groeling, H. A. Weidenmüller, Phys. Reports 299 ('98) 189 [cond-mat/97073]

Standard stat mech: ensemble of *identical* physical systems with *same* Hamiltonian but different initial conditions; Wigner: ensemble of dynamical systems governed by *different* H's with some common symmetry property, seeking generic properties of ensemble due to symmetry.

Dyson, using group-theory results from Wigner, showed 3 generic ensembles:

1) time-reversal invariant with rotational symmetry:

$H_{mn} = H_{nm} = H^*_{mn}$ (orthogonal)

2) time reversal violated (e.g. electron in fixed **B**)

$H_{mn} = H^\dagger_{mn}$ (unitary)

3) time-reversal invariant with 1/2-integer spin & broken rotational symmetry;

$H^{(0)}_{mn}\mathbf{I} - i\Sigma_j H^{(j)}_{mn}\sigma_j$ (symplectic)

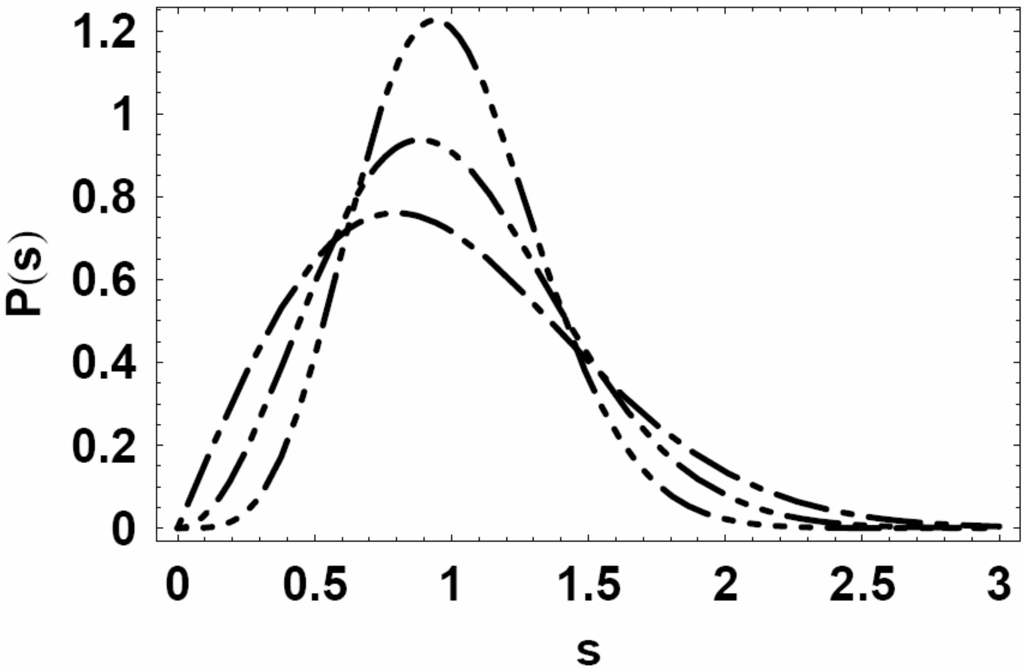
σ_j: Pauli spin matrices, j=1,2,3; **I**: 2 x 2 unit matrix; $H^{()}$ all real, $H^{(0)}$ sym, others asym

Wigner: for convenience, Gaussian weights $P(H) \propto \exp[-(\beta N/\lambda^2) \operatorname{tr} H^2]$

Gaussian **O**rthogonal **E**nsemble: β=1 **GU**nitary**E**: β=2 **GS**ymplectic**E**: β=4

GRMT useless for *average* quantities, but *fluctuations* for large number of levels becomes independent of the form of the level spectrum and of the Gaussian weight factors, and attains

Wigner Surmise (WS)



O $$P_1(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right)$$

U $$P_2(s) = \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi} s^2\right)$$

S $$P_4(s) = \left(\frac{64}{9\pi}\right)^3 s^4 \exp\left(-\frac{64}{9\pi} s^2\right)$$

$$P_\beta(s) = a_\beta s^\beta \exp(-b_\beta s^2)$$

Wigner's argument for the surmise, for the orthogonal ensemble

$p(\mathcal{H}) \sim \exp[-bN\,\mathrm{tr}(\mathcal{H}^2)]$ *N*→∞ ; using Gaussian weighting

Instead, consider *N*=2, orthogonal symmetry: $H = \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix}$

joint probability distribution $p(E_1, E_2)$ for adjacent eigenenergies E_1, E_2 to $P(s)ds$

$\bar{h} \equiv (h_{11} + h_{22})/2$ $u \equiv h_{11} - h_{22}$ $s \equiv \left(u^2 + 4h_{12}^2\right)^{1/2} = |E_2 - E_1|$ so $E_{1,2} = \bar{h} \pm s/2$

Consider all possible MEs h_{11}, h_{22}, h_{12}

$$\iiint p\, dh_{11} dh_{22} dh_{12} = \int ds \underbrace{\iint \exp[-2b(E_1^2 + E_2^2)]\, d\bar{h}\, du \left|\frac{dh_{12}}{ds}\right|}_{p(s)}$$

using $dh_{11} dh_{22} = d\bar{h}\, du$

$$h_{12} = \pm(1/2)(s^2 - u^2)^{1/2} \quad \Rightarrow \quad |dh_{12}/ds| = (s/2)\left(s^2 - u^2\right)^{-1/2}.$$

$$(s/2)\int_{-s}^{s}\left(s^2 - u^2\right)^{-1/2} du = \pi s/2 \qquad 2(E_1^2 + E_2^2) = s^2 + 4\bar{h}^2$$

$$P(s) \sim s\exp(-bs^2)$$

Exact for $N = 2$ and excellent approximation as $N \to \infty$. For large N, the problem of level crossing ($s \to 0$) still reduces to a 2×2 problem near the (usually avoided) degeneracy.
To get $s = 0$, u and h_{12} must vanish simultaneously.

$$p(s) = \int \mathrm{d}u \int \mathrm{d}h_{12}\, p(u, h_{12})\, \delta\Big(s - (u^2 + 4h_{12}^2)^{1/2}\Big) \sim s, \;\; s \ll 1$$

GUE (Gaussian unitary ensemble): H is hermitean, so that h_{12} is complex, and *three* parameters must vanish simultaneously to get $s = 0$:

$$s = \Big[(h_{11} - h_{22})^2 + 4(\Re\mathrm{e}\; h_{12})^2 + 4(\Im\mathrm{m}\; h_{12})^2\Big]^{1/2}$$

Hence, $p(s) \sim s^2$, corresponding to a spherical (rather than circular) shell of radius s in parameter space.

From W. Zwerger, "Theory of Coherent Transport," in T. Dittrich, . . . , W. Zwerger, Quantum Transport and Dissipation (Wiley-VCH, Weinheim, 1998), chap. 1

1957: Wigner surmise for $\beta=1$: $p_1(s) = a_1 s^1 \exp(-b_1 s^2)$, where p is the distribution function of nearest-neighbor energy levels, with s the real spacing over the [local] mean

1960-62: Dyson: circular ensembles: Circular**OE**, **CUE**, **CSE;** NN unitary matrices, eigenvalues $\exp[i\theta_\mu]$, $\mu=1,\ldots N$

N-particle Coulomb gas on a circle (i.e. in 1D), with [shifted] inverse temperature β

Major ingredient: von Neumann–Wigner level repulsion: 2 states connected by a non-vanishing matrix element repel each other—degree of repulsion is determined by symmetry of Hamiltonian—"A simple counting argument leads directly to the exponent $\beta = 1, 2, 4$ in the typical factor $|E_\mu - E_\nu|^\beta$ in the Vandermonde determinant."

Sutherland Hamiltonian for N particles (spinless fermions) on a circle:

$$-\frac{\hbar^2}{2m}\left[\sum_{i=1}^{N}\frac{\partial^2}{\partial\lambda_i^2}-\frac{\beta}{2}(\beta-2)\left(\frac{\pi}{N}\right)^2\sum_{i<j}\frac{1}{\sin^2\left\{\pi\left(\lambda_i-\lambda_j\right)/N\right\}}\right]$$

Application to specific step system: M. Lässig, "Vicinal Surfaces and the Calogero–Sutherland Model," Phys. Rev. Lett. **77** ('96) 526, for Song & Mochrie's observation of tricritical behavior on vicinal Si (113)

OTHER APPLICATIONS of RMT

Localization theory--ensemble of impurity potentials

Clarifies various regimes in mesoscopic physics: clean, ballistic, ergodic, diffusive, critical, localized

Transport in quasi-1D wires

Fluctuations of persistent currents (esp. for non-interacting electrons)

Level spectra of small metallic particles & their response to EM field

Atomic nuclei, atoms and molecules

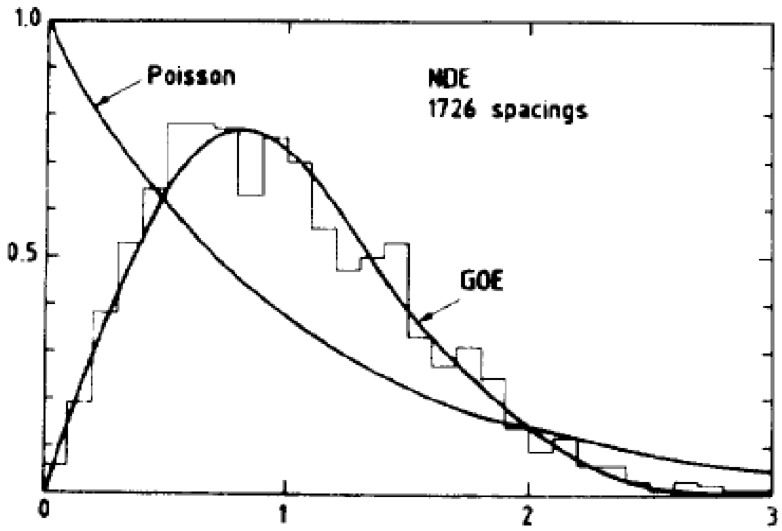
Classical chaos (e.g. Bunimovich stadium, Sinai billiard)

QCD, supersymmetry

2D quantum gravity

Examples of NN spacing distributions with GOE (ϱ =1)

Fig. 1. Nearest-neighbor spacing distribution for the "Nuclear Data Ensemble" comprising 1726 spacings (histogram) versus $s = S/D$ with D the mean level spacing and S the actual spacing. For comparison, the RMT prediction labelled GOE and the result for a Poisson distribution are also shown as solid lines. Taken from Ref. [1].



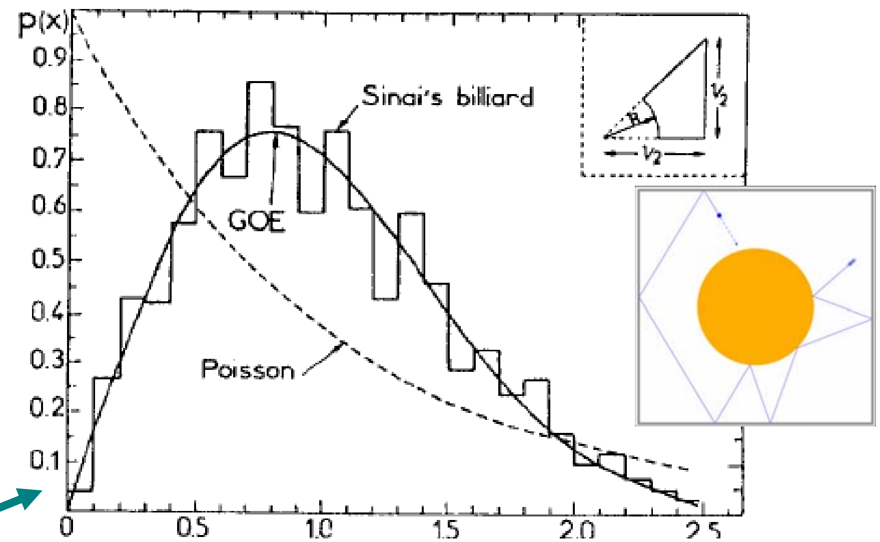


Fig. 4. The nearest-neighbor spacing distribution versus s (defined as in Fig. 1) for the Sinai billiard. The histogram comprises about 1000 consecutive eigenvalues. Taken from Ref. [5].

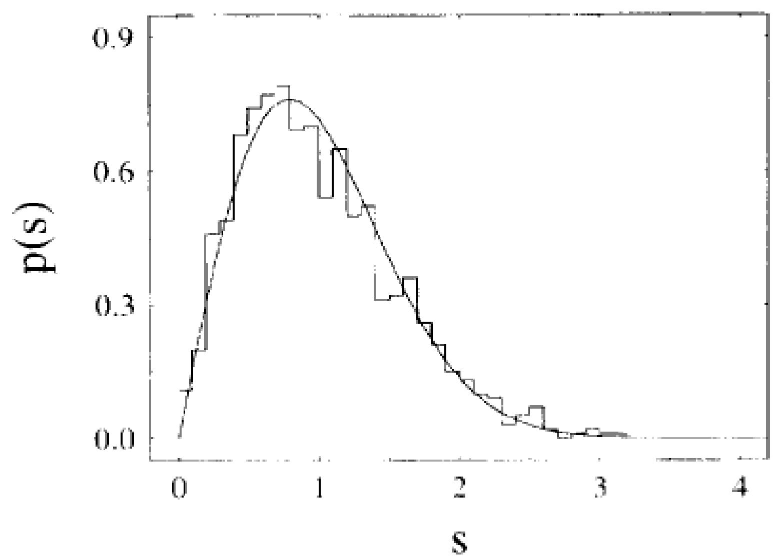
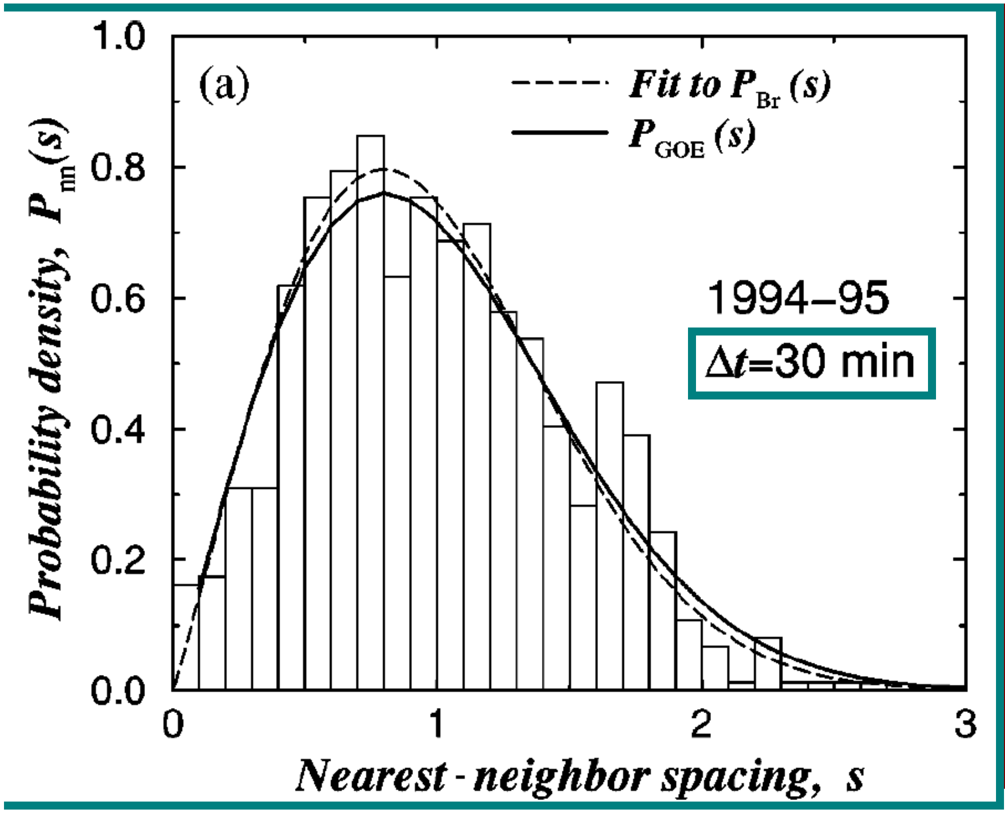


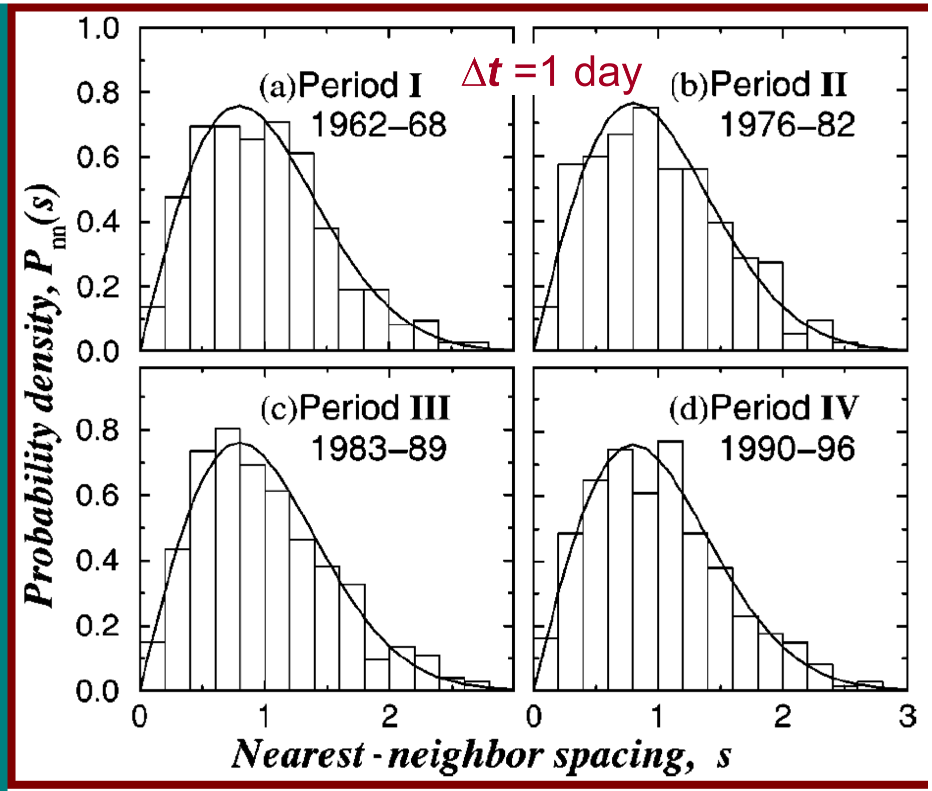
Fig. 6. Nearest-neighbor spacing distribution for elastomechanical modes in an irregularly shaped quartz crystal.

T. Guhr et al. / Physics Reports 299 (1998) 189

RMT & financial data: Cross-correlations of price fluctuations of different stocks, using $P_1(s)$

V. Plerou, …, T. Guhr, and H. E. Stanley, PRE 66 ('02) 066126

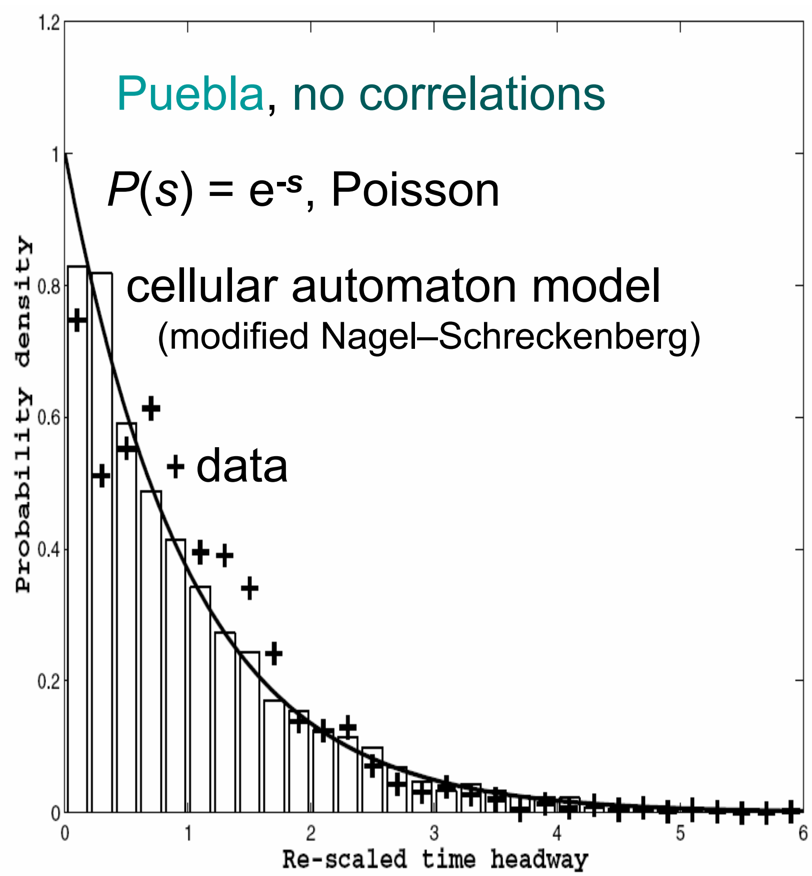


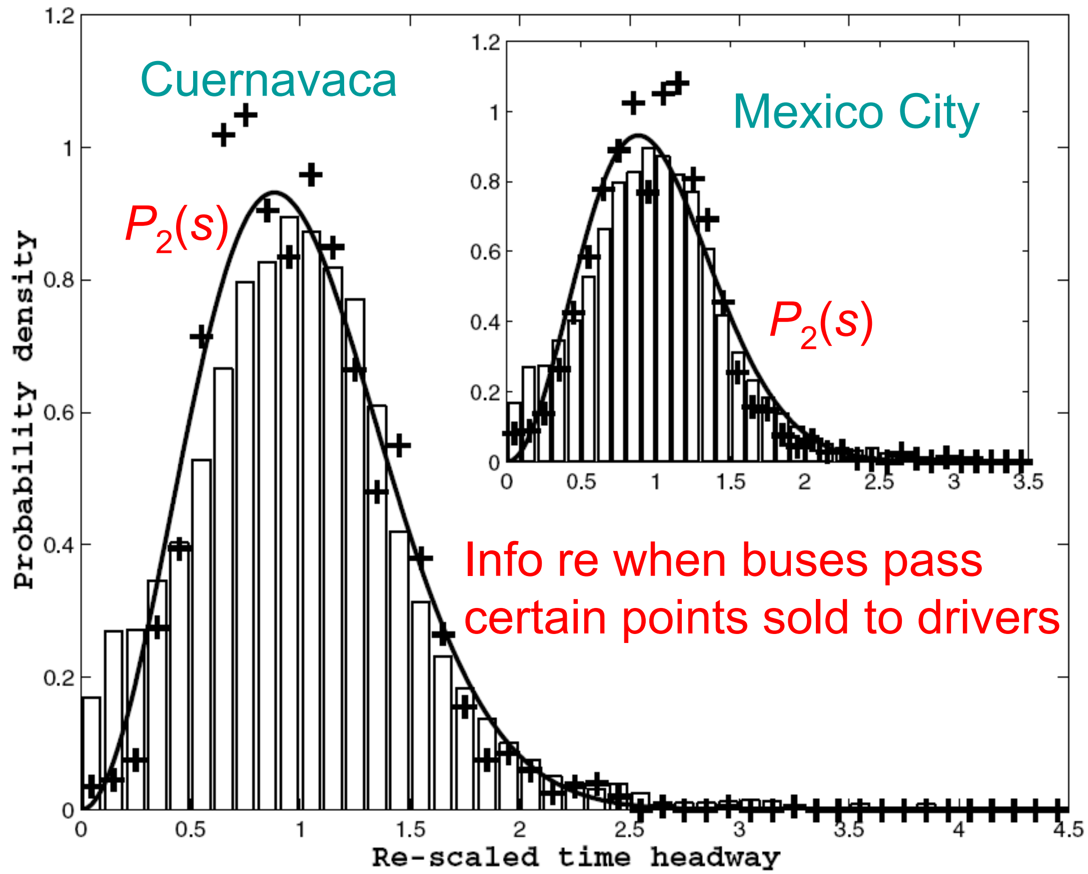


Headway statistics of buses in Mexican cities, using $P_2(s)$

M. Krbálek & P. Šeba, J. Phys. A **36** ('03) L7; **33** ('00) L229

Headway: time interval Δt between bus and next bus passing the same point
No timetable for buses in Mexico; independent drivers seek to optimize # riders/fares

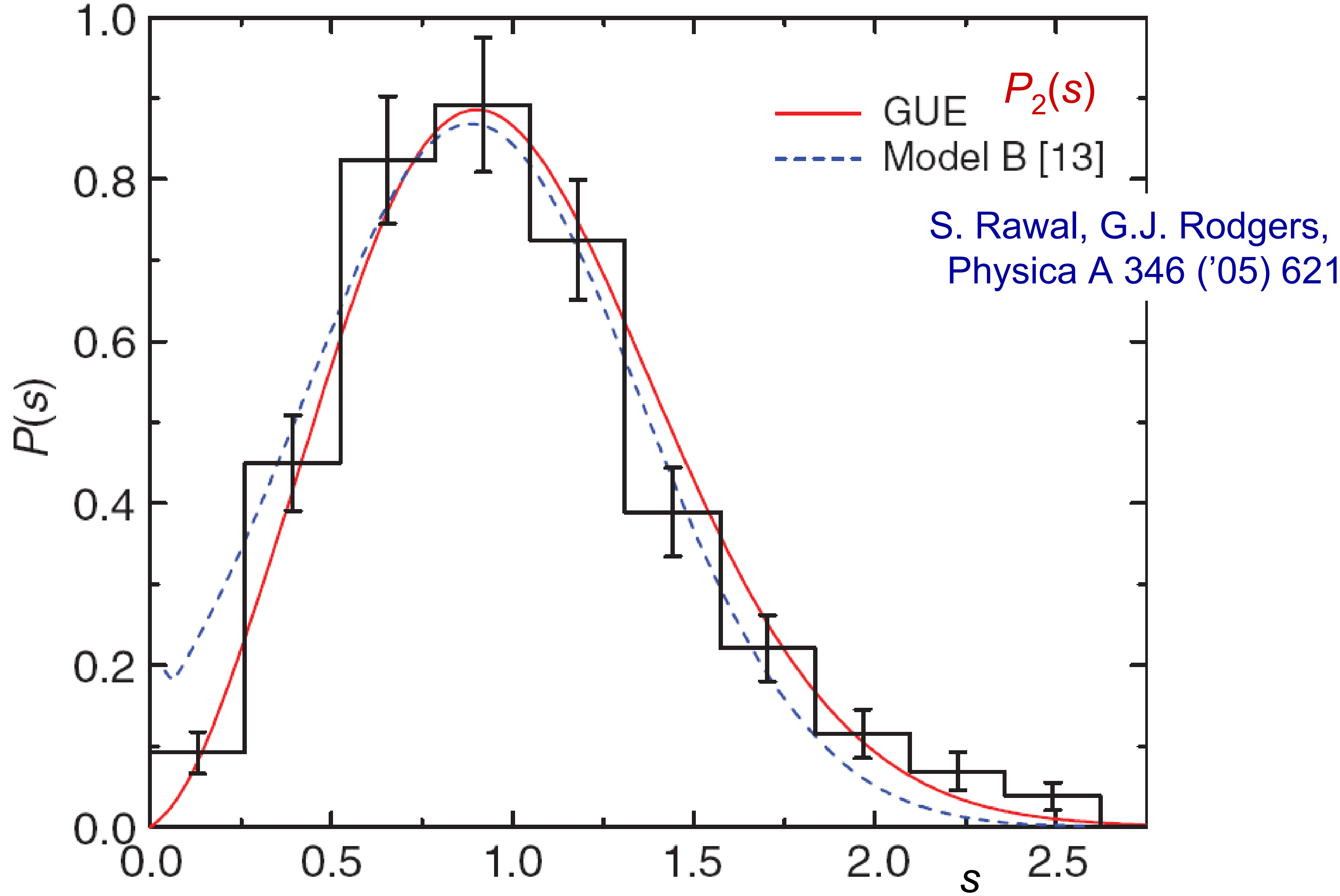




WS $P_2(s)$ better than CA because in CA, correlations only between NNs

Modelling gap-size distribution of parked cars using RMT

A.Y. Abul-Magd, Physica A 368 ('06) 536



Unlike random sequential process, Coulomb gas extends repulsion beyond geometric size.

What about ϱ other than 1,2,4?

Mixed states of different symmetry; Brody distribution, etc.

Recent noteworthy illustration: zeroes of Riemann ζ
A. Pimpinelli, J. Phys. A, in press

$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z}$$

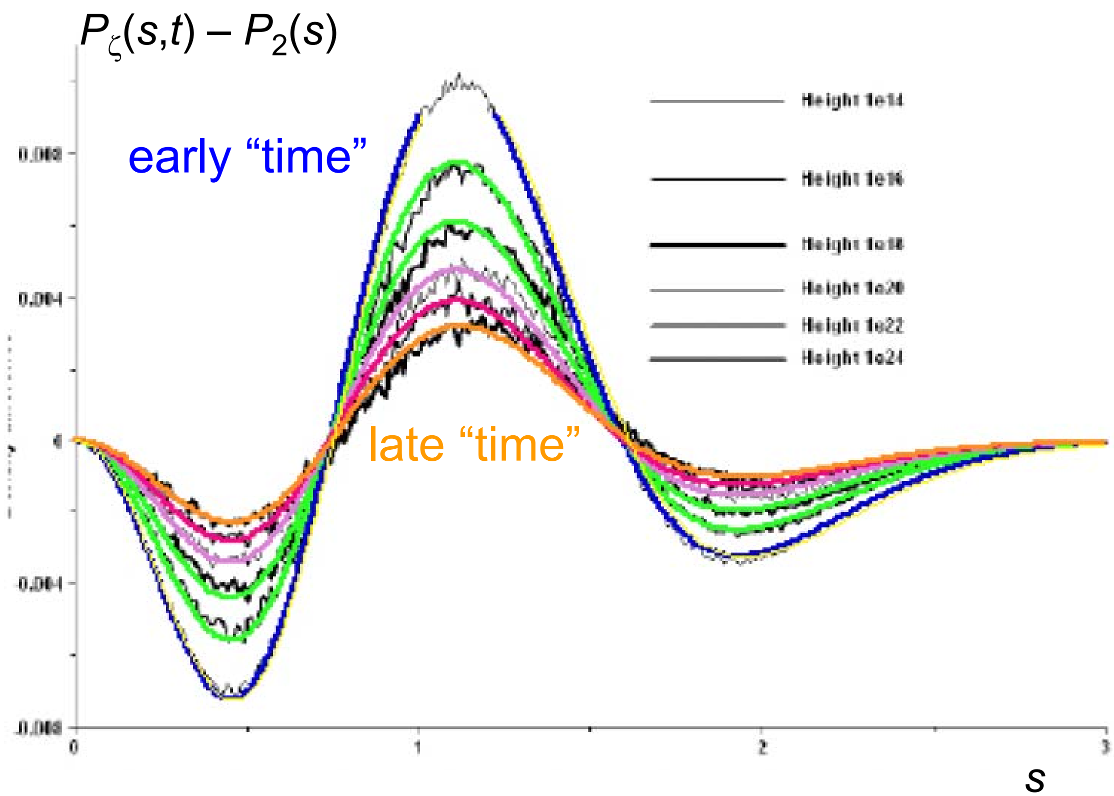
Zeroes $\zeta(z_n)$ at $z_n = 1/2 + iT_n$

$$t = \sqrt{\log_{10} T}$$

$s \propto z_{m+1} - z_m$ starting at t

Conjecture:

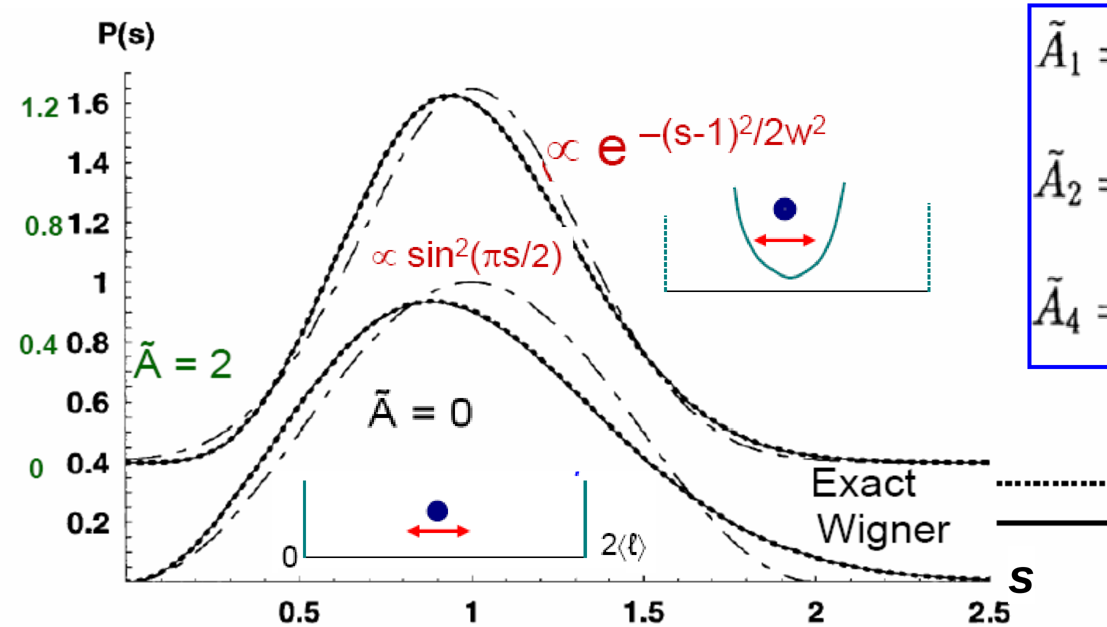
$$P_\zeta(s, t(T)) = P_{GSE+GUE \to GUE}(s, t(T))$$



$$P_\zeta(s, t(T)) = P_{GSE+GUE \to GUE}(s, t(T)) = P_{MF}(s,t) + O(\exp(-2t))$$

$$\approx P_2(s) + \kappa\{[P_4(s) - P_2(s)]\exp(-t)\} + O(\exp(-2t))$$

Wigner Surmise (WS) for TWD (terrace-width distribution)



$\tilde{A}_1 = -1/4: \quad P_1(s) = \frac{\pi}{2} s \exp\left(-\frac{\pi}{4}s^2\right)$

$\tilde{A}_2 = 0: \quad P_2(s) = \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi}s^2\right)$

$\tilde{A}_4 = 2: \quad P_4(s) = \left(\frac{64}{9\pi}\right)^3 s^4 \exp\left(-\frac{64}{9\pi}s^2\right)$

$U(\ell) = A/\ell^2$

$$\tilde{A} \equiv \frac{\tilde{\beta} A}{(k_B T)^2}$$

Generalizing from the special cases:

WS → GWS

- The three special cases correspond to $\varrho = 1$, 2, and 4.
- $\tilde{A}$ and ϱ are related by: $\tilde{A} = (\varrho - 2)\varrho/4$; $\varrho = 1 + \sqrt{1 + 4\tilde{A}}$
- Simplest interpolation expression: $P_\varrho(s) = a_\varrho s^\varrho \exp(-b_\varrho s^2)$
- Two conditions on $P_\varrho(s)$: normalization & unit mean
  $\Rightarrow$ values of a_ϱ, b_ϱ (in terms of Γ functions).

Comparison of variance of $P(s)$ vs. Ã computed with Monte Carlo:

GWS does better, quantitatively & conceptually, than any other approximation

Hailu Gebremariam et al., Phys. Rev. B 69 ('04)125404

Experiments measuring variances of TWDs

Vicinal	T (K)	σ^2	ϱ	$\tilde{A}$	A_W/A_G	A_W (eV Å)	Experimenters
Pt(1 1 0)-(1 × 2)	298		2.2	0.13	–	$\tilde{\beta}=?$	Swamy, Bertel [36]
Cu(19, 17, 17)	353	0.122	4.1	2.2	0.77	0.005	Geisen [5,54]
Si(1 1 1)	1173	0.11	3.8	1.7	0.96	0.4	Bermond, Métois [55]
Cu(1,1,13)	348	0.091	4.8	3.0	1.27	0.007	Giesen [5,56]
Cu(1 1,7,7)	306	0.085	5.1	4	1.37	0.004	Geisen [5,54]
Cu(1 1 1)	313	0.084	5.0	3.6	1.39	0.004	Geisen [5,54]
Cu(1 1 1)	301	0.073	6.0	6.0	1.58	0.006	Geisen [5,54]
Ag(1 0 0)	300	0.073	6.4	6.9	1.58	$\tilde{\beta}=?$	P. Wang...Williams
Cu(1, 1, 19)	320	0.070	6.7	7.9	1.64	0.012	Geisen [5,56]
Si(1 1 1)-(7 × 7)	1100	0.068	6.4	7.0	1.67	0.7	Williams [57]
Si(1 1 1)-(1 × 1)Br	853	0.068	6.4	7.0	1.67	0.1	X.-S. Wang, Williams [58]
Si(1 1 1)-Ga	823	0.068	6.6	7.6	1.67	1.8	Fujita...Ichikawa [59]
Si(1 1 1)-Al $\sqrt{3}$	1040	0.058	7.6	10.5	1.85	2.2	Schwennicke...Williams [60]
Cu(1, 1, 11)	300	0.053	8.7	15	1.95	0.02	Barbier et al. [21]
Cu(1, 1, 13)	285	0.044	10	20	2.12	0.02	Geisen [5,56]
Pt(1 1 1)	900	0.020	24	135	2.59	6	Hahn...Kern [61]
Si(1 1 3) rotated	1200	0.004	124	3.8×10^3	2.92	$(27\pm5)\times10^2$	van Dijken, Zandvliet, Poelsema [9]

Why Look for Fokker-Planck Equation for TWD?

- Justification/derivation of generalized continuum Wigner surmise (beyond H_{eff} of Richards et al.) since no symmetry basis for $\varrho \neq 1, 2,$ or 4
- Dynamics: how non-equilibrium TWD (e.g. step bunch) evolves toward equilibrium
- Quench or upquench: sudden change of T does not change A much but changes $\tilde{A}$ (and so ϱ) considerably
- Connections with other problems, e.g. capture zone distribution (& Heston model of econophysics)

Derivation of Fokker-Planck for TWD

- Start with Dyson Coulomb gas/Brownian motion model: repulsions $\propto$ 1/(separation) & parabolic well

$$\dot{x}_i = -\gamma x_i + \sum_{i \neq j} \frac{\hat{\varrho}}{x_i - x_j} + \sqrt{\Gamma}\eta$$

- Assume steps beyond nearest neighbors are at integer times mean spacing (cf. Gruber-Mullins)

  $\dot{s} = -\kappa s + \rho/s +$ noise

$$\tilde{t} \equiv t\Gamma / \langle \ell \rangle^2$$

1/τ

Noise sets time scale.

- Demand self-consistency for width of parabolic confining well: $\kappa \rightarrow 2b_\rho$

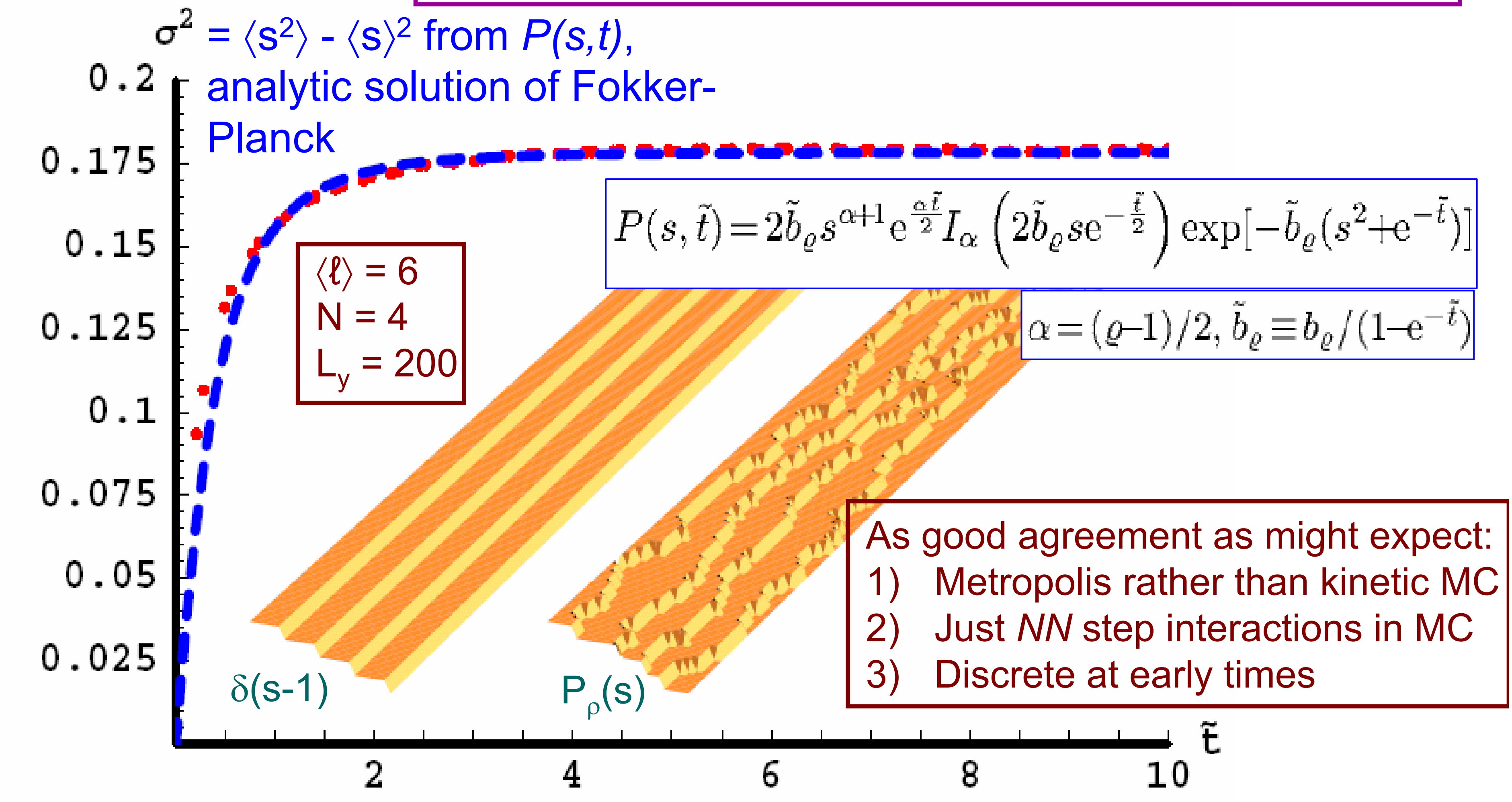
$$\frac{\partial P(s,\tilde{t})}{\partial \tilde{t}} = \frac{\partial}{\partial s}\left[\left(2b_\varrho s - \frac{\varrho}{s}\right)P(s,\tilde{t})\right] + \frac{\partial^2}{\partial s^2}[P(s,\tilde{t})] \quad \rightarrow P_\varrho(s)$$

Check of Fokker-Planck with Monte Carlo

cleaved → equilibrium

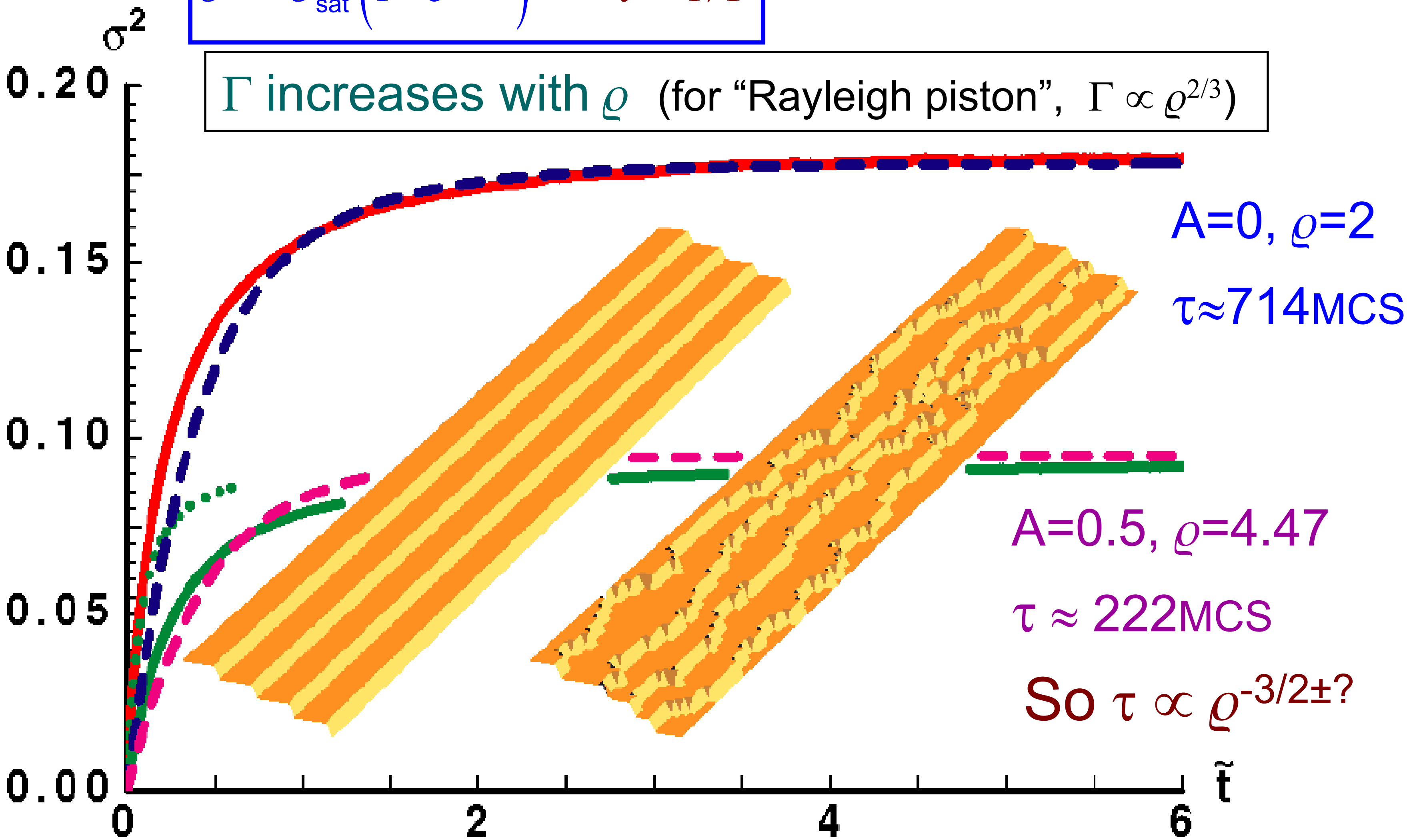
TSK model (no adatom carriers)

Best match for 1.4 FP time units = 10^3 MCS



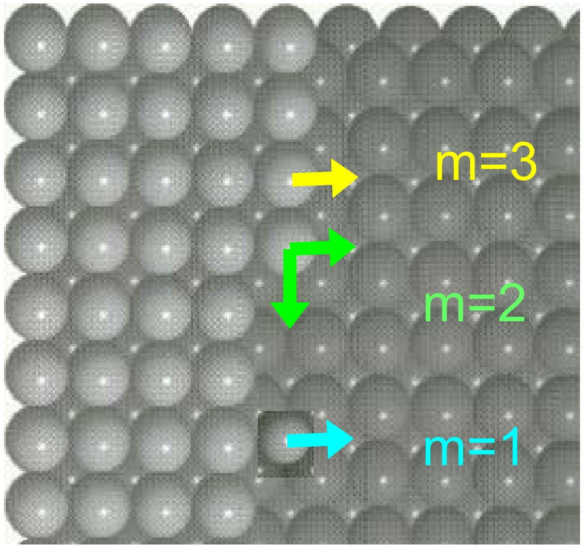
Fokker-Planck vs. Monte Carlo: Effect of Step-step Repulsions

$$\sigma^2 \approx \sigma_{\text{sat}}^2 \left(1 - e^{-t/\tau}\right) \qquad \tau \propto 1/\Gamma$$



Qualitative result: τ decreases as repulsion rises

Improved tests: Kinetic MC & SOS model

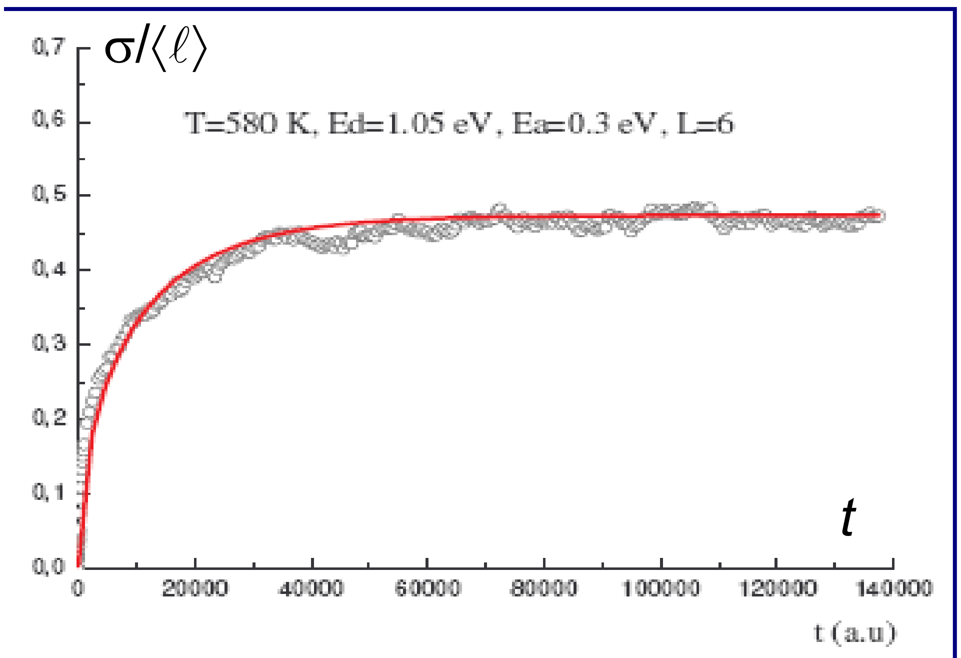


$E_{barrier} = E_d + m\, E_a$ breaking m bonds

E_d = 0.9 -1.1 eV; E_a = 0.3 - 0.4 eV

T = 520 - 580 K

$\langle \ell \rangle$ = 4-15, 5 steps, 10000 x L_x



Fit:

$$\sigma(t) = \sigma_{sat}\sqrt{1-\exp(-t/\tau)}$$

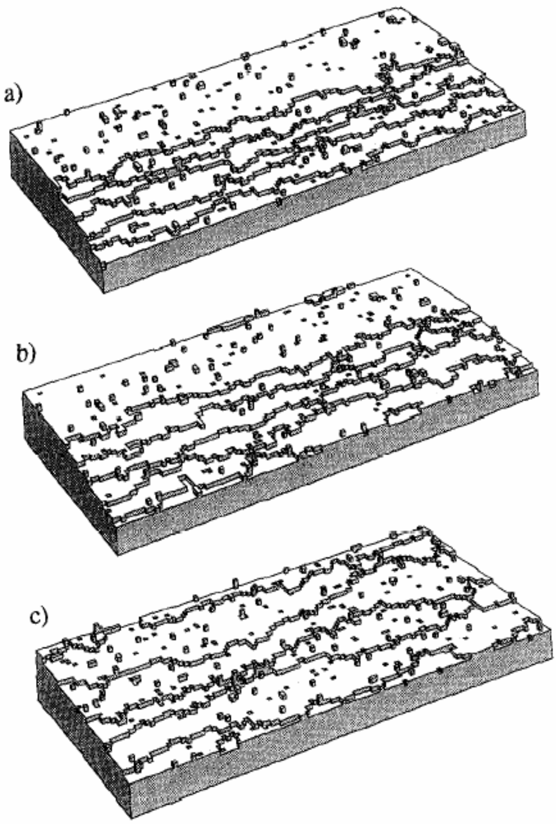
Expect $\tau \propto \exp(E_{barrier}/k_B T)$

Find $E_{barrier} \approx 1\, E_d + 3\, E_a$

Behavior of τ in SOS via KMC: Ramp E_d, E_a, T, $\langle \ell \rangle$

Unpublished; please write for preprint!

2 other situations of interest



Step Bunch: initially a delta function

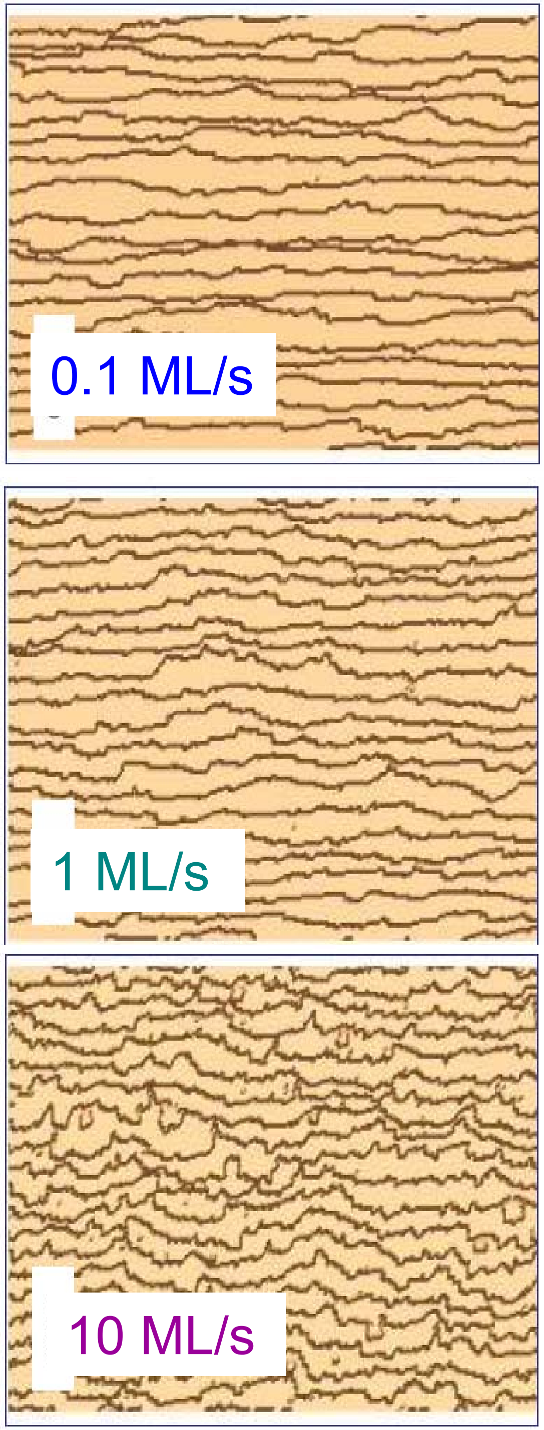
$$P(s,\tilde{t}) \to \frac{a_\varrho s^\varrho}{(1-\mathrm{e}^{-\tilde{t}})^{(\varrho+1)/2}} \exp[-s^2 b_\varrho/(1-\mathrm{e}^{-\tilde{t}})]$$

Quench or upquench: change from initial ρ_0 to ρ, e.g. change in temperature

$$P(s,\tilde{t}) = a_\varrho s^\varrho \mathrm{e}^{-\tilde{b}_\varrho s^2} \frac{(1-\mathrm{e}^{-\tilde{t}})^{\frac{\varrho_0-\varrho}{2}}}{(1-\mathrm{e}^{-\tilde{t}}(1-b_\varrho/b_{\varrho_0}))^{\frac{\varrho_0+1}{2}}} \, {}_1F_1\left(\frac{\varrho_0+1}{2}, \frac{\varrho+1}{2}, \frac{\tilde{b}_\varrho s^2}{1+(b_{\varrho_0}/b_\varrho)(\mathrm{e}^{\tilde{t}}-1)}\right)$$

Final

Does growth flux (step motion) alter TWD?



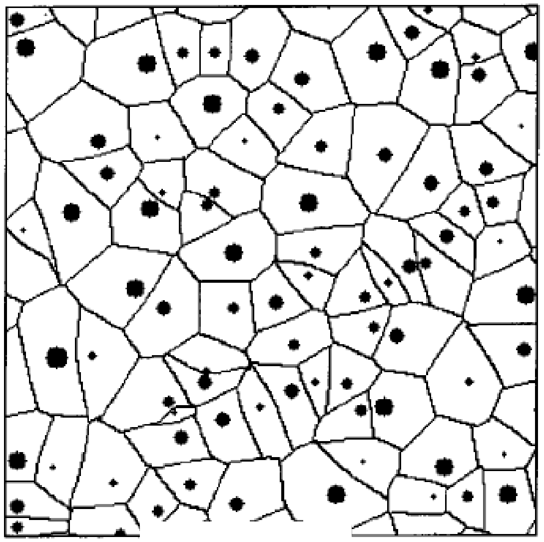
Test: *no* energetic interaction (ϱ=2), 150 ML

Unpublished; please write for preprint!

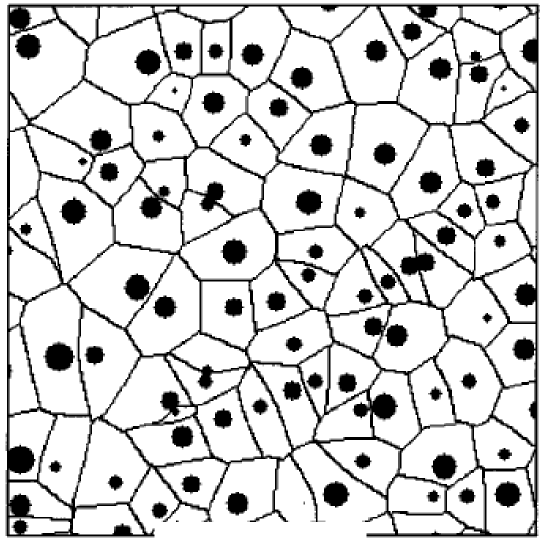
- Narrower ⇒ *effective* repulsion that rises with flux, higher ϱ, more Gaussian-like
- Decreased apparent stiffness $\tilde{\beta}$

Evolution of Island Structures: Simulations of i=1

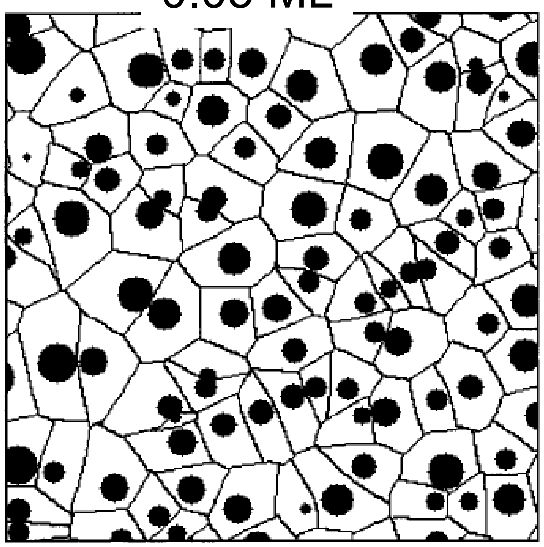
Circular Islands Mulheran & Blackman, PRB 53 (96) 10261



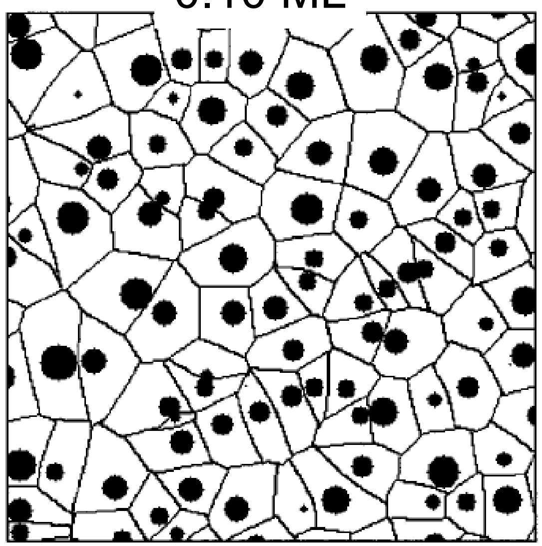
0.05 ML



0.10 ML

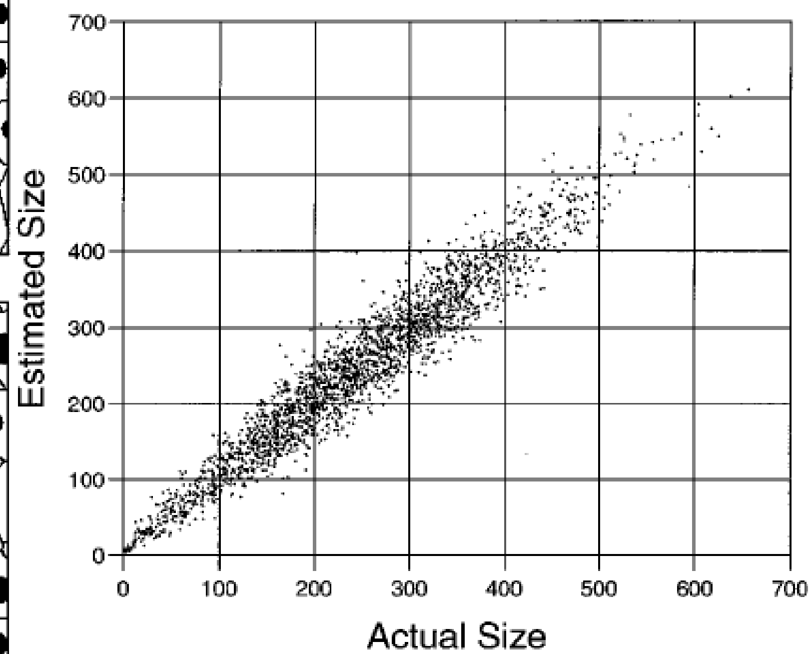


0.15 ML



0.20 ML

Estimated size of island based on Voronoi polygon CZ ∝ actual size of island



Island Size Scaling, stable config *i*

Amar & Family, PRL 74 (95) 2066

Dynamic scaling assumption

$$N_s(\theta) = \theta S^{-2} f_i(s/S)$$

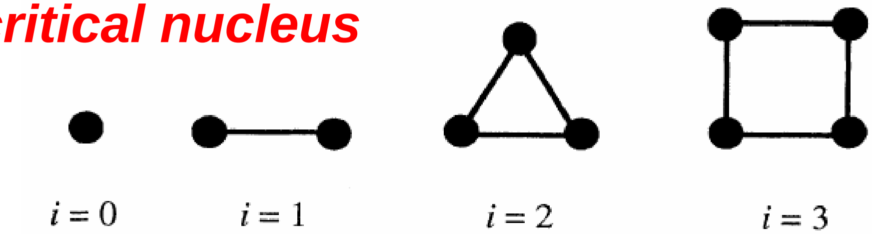
Bartelt & Evans

$$f_i(u) = C_i u^i e^{-i a_i u^{1/a_i}}$$

$$\frac{\Gamma[(i+2)a_i]}{\Gamma[(i+1)a_i]} = (i a_i)^{a_i}, \quad C_i = \frac{(i a_i)^{(i+1)a_i}}{a_i \Gamma[(i+1)a_i]}$$

***i*+1 atoms: smallest stable island**

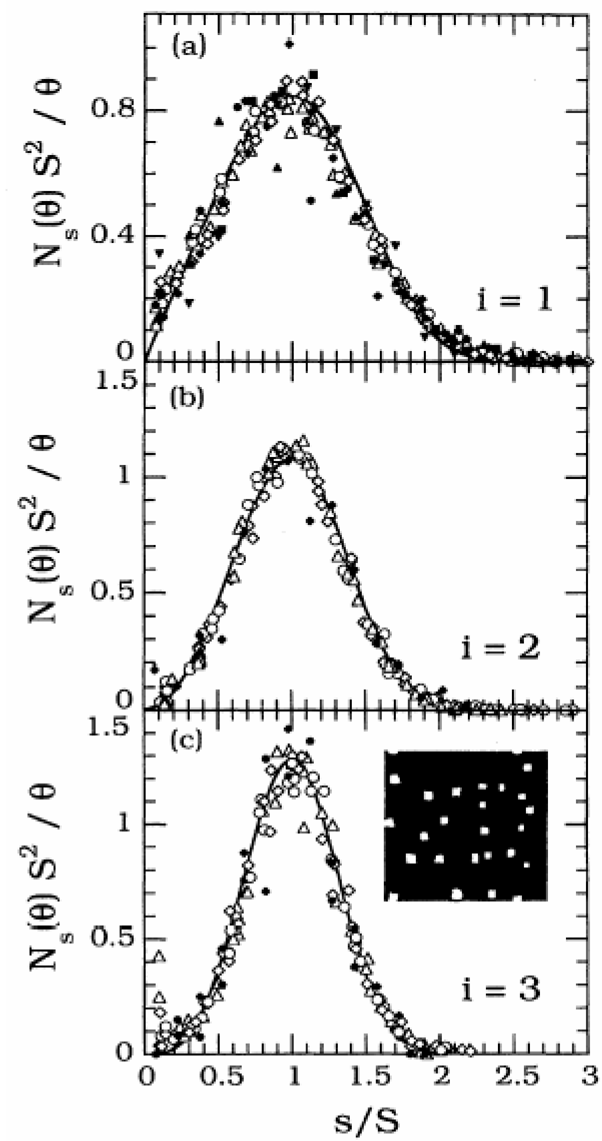
critical nucleus



In contrast to Point-Island Rate Eqn for large D/F

$$f_i(u) = \frac{1}{i+2}\left(1 - \frac{i+1}{i+2} u\right)^{-\frac{i}{i+1}}; \quad 0 \le u \le \frac{i+2}{i+1}$$

$$f_i(u) = 0; \quad u > \frac{i+2}{i+1}$$



Scaling During Growth in 1D: Going Beyond Mean-Field Rate Eqns.

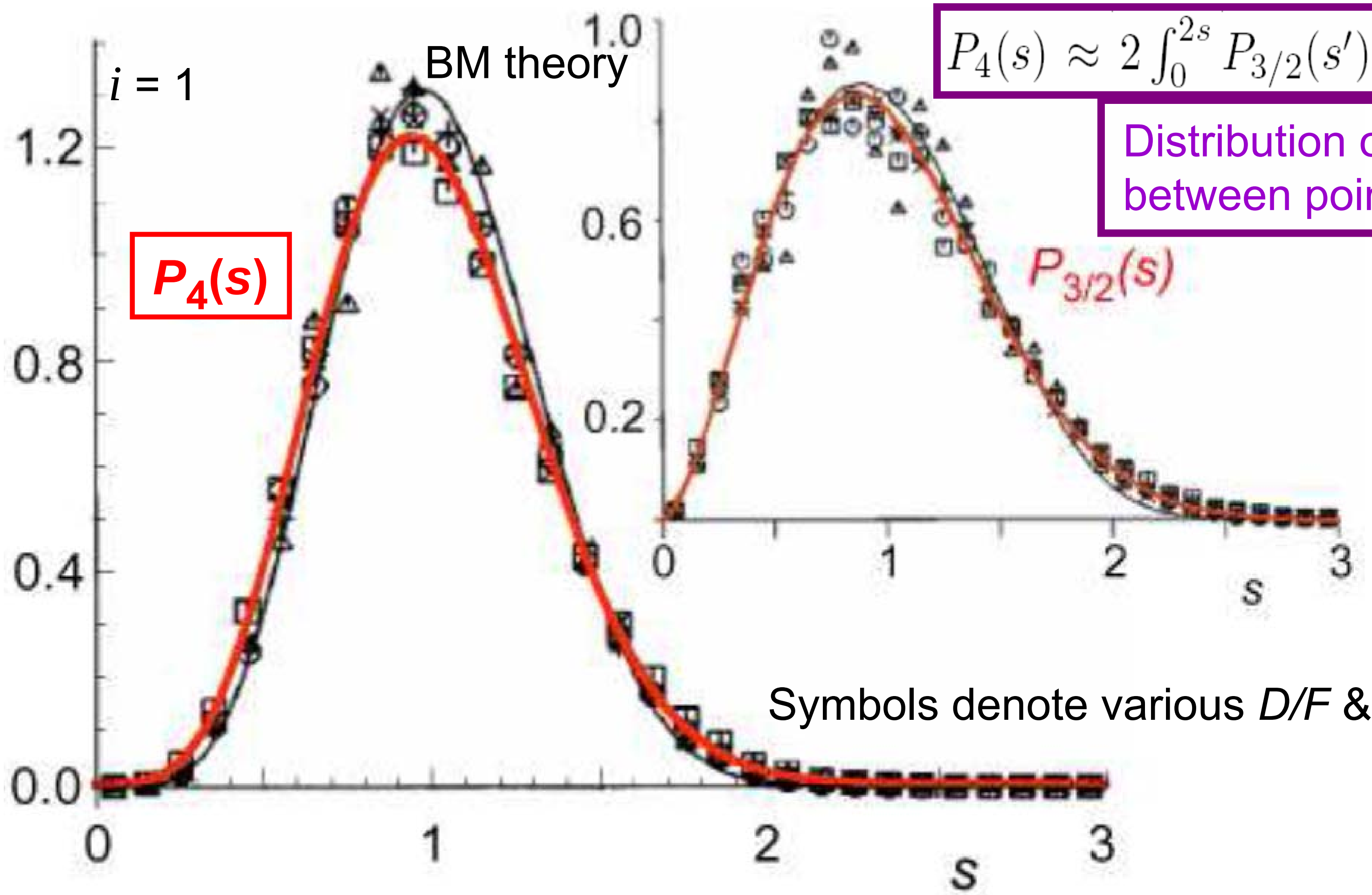
Blackman & Mulheran, PRB 54 (96) 11681

$P_4(s)$ fits numerical data at least as well as B&M's complicated theory expression (not expressible succinctly)

$$d = 1 \Rightarrow \varrho = 2(i + 1)$$

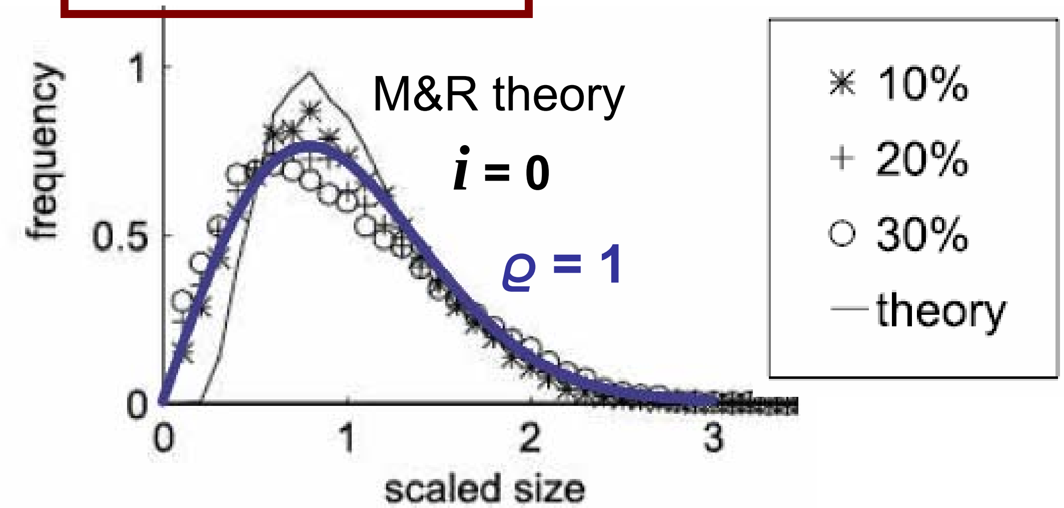
$$P_4(s) \approx 2\int_0^{2s} P_{3/2}(s')P_{3/2}(2s-s')\,ds'$$

Distribution of gaps between point islands

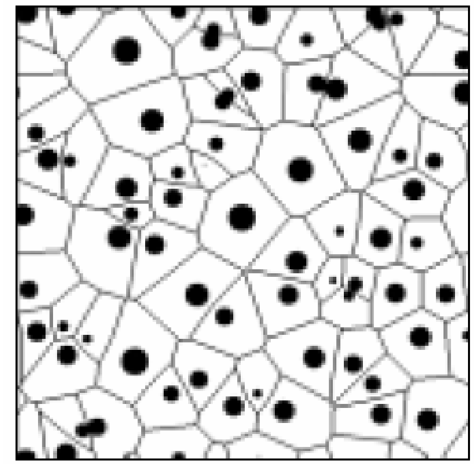


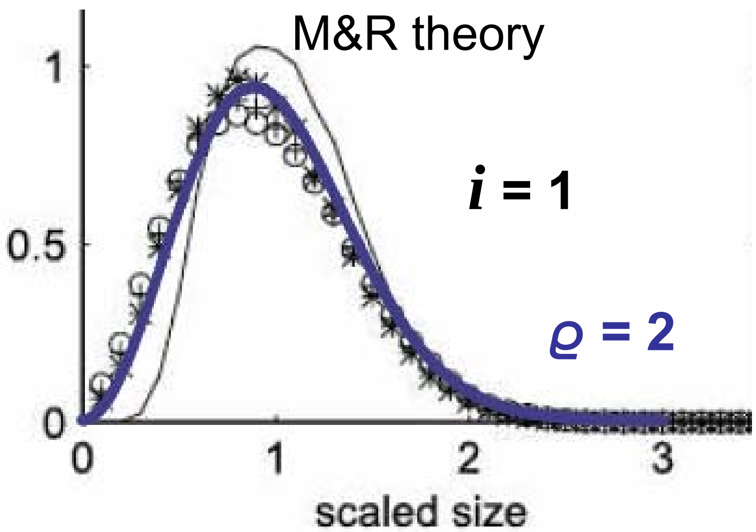
Theory of CZ size distributions in growth, *Mulheran & Robbie*, EPL 49(00)617

$d = 2 \Rightarrow \varrho = i + 1$

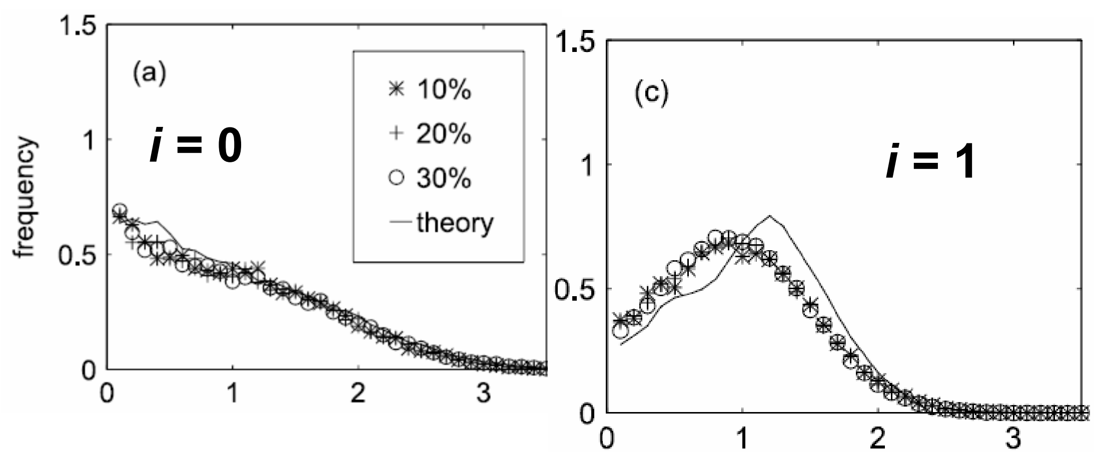


Wigner distribution $P_{\varrho}(s)$ fits much better than M&R theory



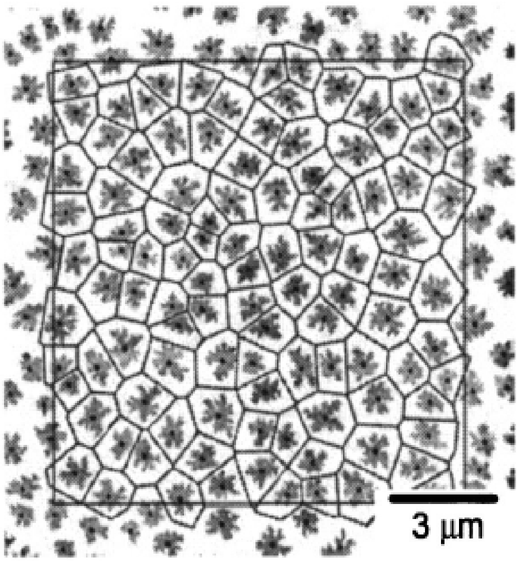


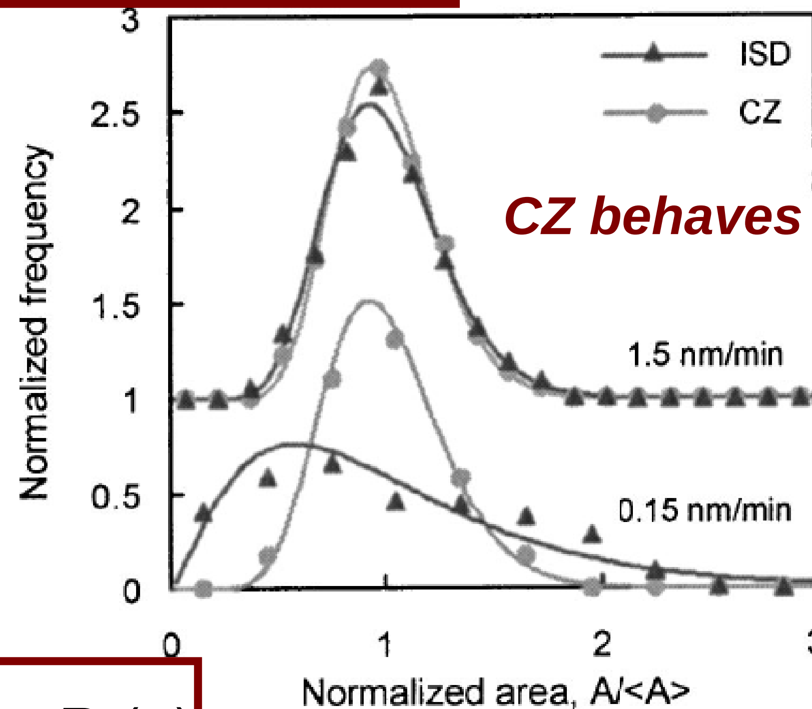
Island size distribution not so informative



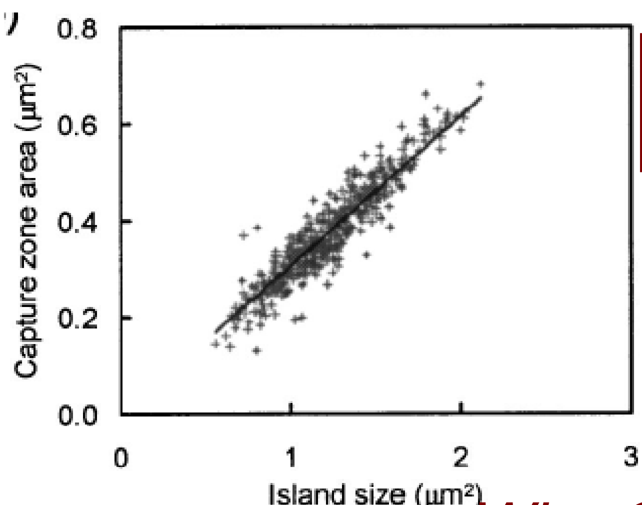
Exp't: Pentacene/SiO_2 Pratontop et al.,PRB 69 (04) 165201

Gamma func'n $\Pi_\alpha(x) = [\alpha^\alpha / \Gamma(\alpha)] \; x^{\alpha-1} \exp(-\alpha x)$



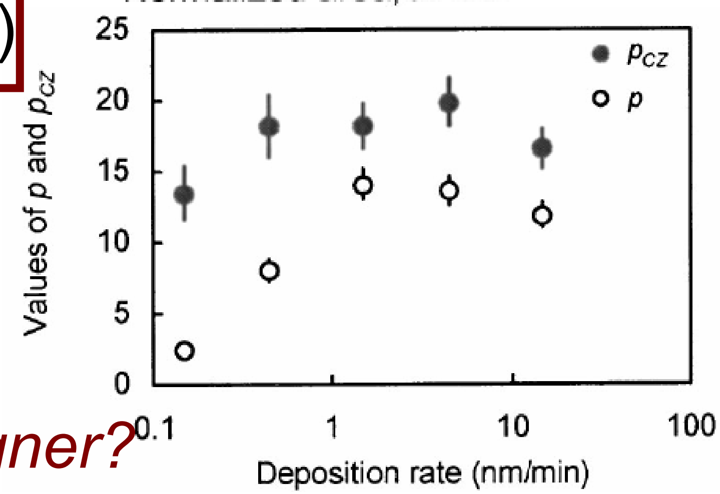


CZ behaves better



$\Pi_{2\varrho+\alpha_0}(s) \approx P_\varrho(s)$

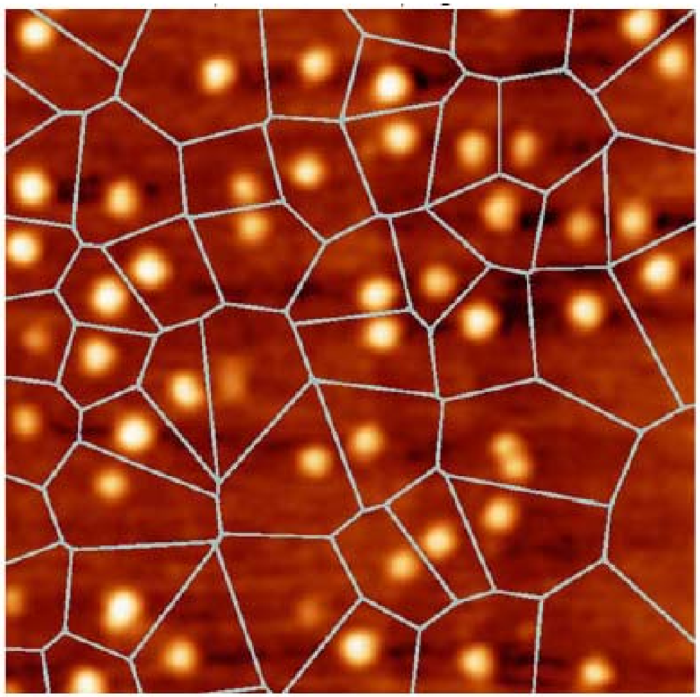
but Π more skewed



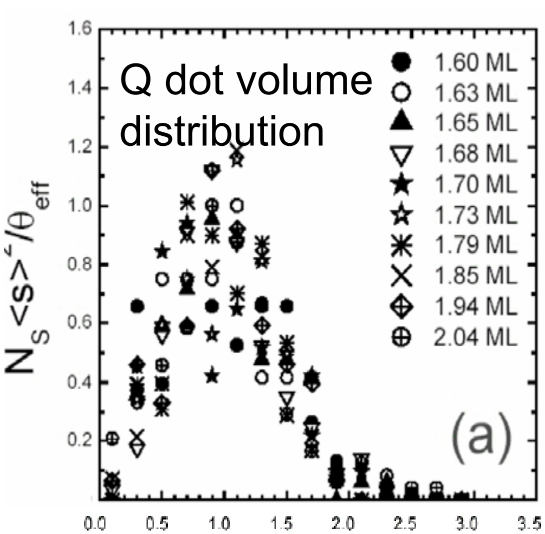
Why Gamma, not Wigner?

Scale invariance in thin film growth: InAs *quantum dots* on GaAs(001)

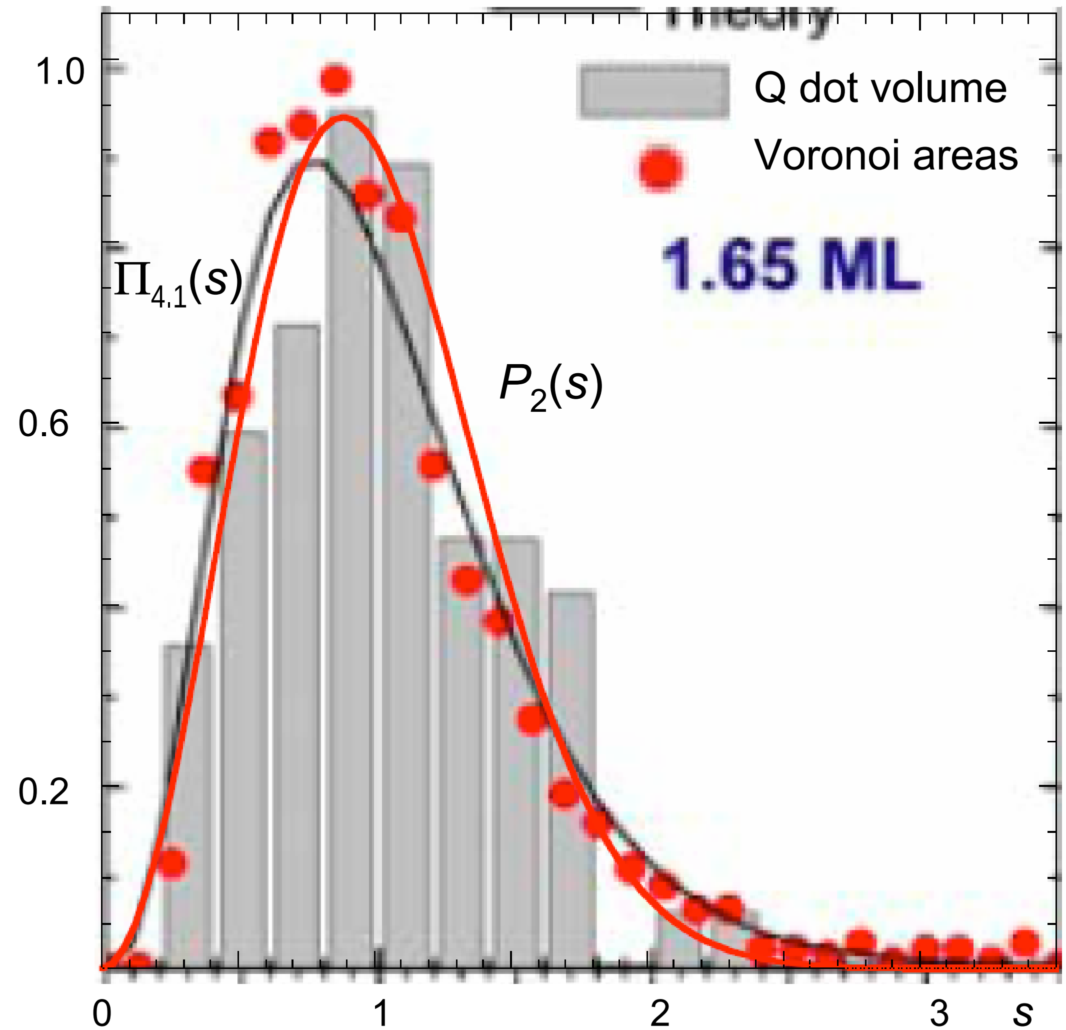
M. Fanfoni *et al.*, PRB **75** (' 07) xxx



AFM, 1.68 ML, 350x350nm^2, 500°C



Θ (ML)	1.65	1.68	1.70	1.73	1.79	1.85
α	4.1	4.6	4.5	4.7	4.5	4.6



Why it works: Phenomenological theory

CZ does "random walk" with 2 competing effects on *ds/dt*:

1] Neighboring CZs hinder growth ⇒ external pressure, repulsion *B* leads to force $-KBs$ Also noise η

2] Non-symmetric confining potential, new island nucleated with large size so force stops fluctuations of CZ to tiny values
In Dyson model, logarithmic interaction, so $+K\,(\ \ )\,/s$

3] Can argue in 2D that ( ) is $i + 1$
using critical density $\propto s^i$, # sites visited in lifetime $\propto s^1$
entropy $\propto$ - product s^{i+1}, & force $-\partial$ (entropy) / ∂s
[Also $i + 1$ in 3D & 4D; but $2(i + 1)$ in 1D]

$\acute{N} = \sigma n N_i = \sigma n^{i+1}$
$\sigma = D/\ell^{2-d}$ $\quad s \equiv \ell^d$
$n \propto \ell^2 \approx s^{2/d}$
prod $\propto s^{(2/d)(i+1)}$

4] Combine ⇒ Langevin eq. $ds/dt = K\left[(2/d)(i+1)/s - Bs\right] + \eta$ [d=1,2]

5] Leads to Fokker-Planck eq. with stationary sol'n $P_\varrho(s)$
cf. AP, HG, & TLE, Phys. Rev. Lett. **95** (05) 246101

Summary (see http://www2.physics.umd.edu/~einstein)

- TWD of vicinals provides physical entrée to intriguing 1D fermion models & RMT, can connections to many other current physics issues---universality in fluctuations
- Generalized Wigner surmise (GWS) relevant to problems in many fields, with ϱ having physical meaning
- For TWD, $\varrho = 1 + [1 + 4 A\tilde{\beta}/(k_B T)^2]^{1/2}$
- With Fokker-Planck, study relaxation
- Narrowing of TWD due to growth
- Look at distribution of areas of capture zones, rather than island sizes
- CZ well described by GWS $P_\varrho(s)$, characteristic of universal fluctuations, with $\boldsymbol{\varrho = (2/d)\,(i + 1)}$

References (download: http://www2.physics.umd.edu/~einstein)

- *Capture-Zone Scaling in Island Nucleation: Phenomenological Theory Linking to Random-Matrix Theory*, Alberto Pimpinelli & TLE, submitted
- *Fokker-Planck Approach to Relaxation of Terrace-width Distributions on Vicinals: Physical Information in the Time Constant*, Ajmi Bhadj-Hamouda, Alberto Pimpinelli, and TLE, submitted
- *Using the Wigner-Ibach Surmise to Analyze Terrace-Width Distributions: History, User's Guide, and Advances*, TLE, Appl. Phys. A 87, 375 (2007)
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- *Distribution of Terrace Widths on a Vicinal Surface in the One-Dimensional Free-Fermion Model*, B. Joós, TLE, and N. C. Bartelt, Phys. Rev. B 43, 8153–8162 (1991)