Pimpinelli and Einstein Reply: In [1], we proposed analyzing the capture-zone (CZ) distribution of islands in submonolayer epitaxial growth by fitting with the generalized Wigner surmise (GWS) [2]: $P_\beta(s) = a_\beta s^\beta \exp(-b_\beta s^2)$; s is the CZ area A over its mean $\langle A\rangle$, and β is the sole adjustable parameter. Our mean-field (MF) argument for $P_\beta(s)$ also suggested that β was the size of the smallest stable nucleus of an island, $i+1$ (i.e., i is the critical nucleus), in dimensions $d \geq 2$, and $2(i+1)$ in $1d$. $P_\beta(s)$ fits experimental data at least as well as the alternatives. Furthermore, much (but not all) Monte Carlo data supported the deduced value of β in terms of i for $1d$ and $2d$. However, more thorough analysis and numerical testing was clearly warranted.

Recently, Amar's [3] and Evans's groups [4] [SSA and LHE, respectively] have taken up this challenge and produced extensive numerical data, SSA for two models of point islands in $d = 1$, 2, 3, 4 [5], and LHE for compact islands in $2d$, the case more appropriate for comparison with experiment. Space limits our focus here to $2d$. Both groups' results differ notably from our MF description, arguably reminiscent of using mean field for critical phenomena. Specifically, with $i = 1$ and fractional coverage $\theta = 0.1$, SSA found for both point-island models that β was closer to 3 than our MF-predicted $\beta = 2$. Up to $\theta \geq 0.4$, β did not change with θ, but β decreased modestly as D/F, the ratio of the rates for atom hopping and for deposition, ramped up over 10^5–10^{10}, reaching $\beta \approx 2.8$ as $D/F \to \infty$ [6].

For compact islands with $i = 1$, LHE's data is likewise better described by $\beta \approx 3$ than 2—cf. Fig. 1. Also, the variance is that of a GWS with $\beta = 2.97$. LHE's data for $i = 0$ is even closer to $\beta = 2$, and the variance yields $\beta = 1.90$. Both SSA and LHE find $\beta \approx i + 2$ accounts for the data better than $i + 1$. However, the distribution is more skewed than $P_\beta(s)$. LHE find the optimal fit occurs with a distribution between GWS and the oft-used gamma distribution $G_\alpha(s)$ [7]. The log-log plot in their Fig. 1 suppresses this exponential factor for small s; their plot supports $\beta \approx 4$. We advocate emphasizing data near the peak, where the count rate is highest and the fractional error is smallest. This procedure is especially warranted when dealing with experimental data, in which the number of CZs is 2–3 orders of magnitude smaller than in these simulations. Figure 1 shows that $\beta = 3$ describes the overall data better than $\beta = 4$, especially regarding width and peak height [6]. Fits with $P_3(s)$ and $G_7(s)$ are comparable [as are fits of LHE's unpublished data for $i = 0$ by $P_2(s)$ and $G_5(s)$].

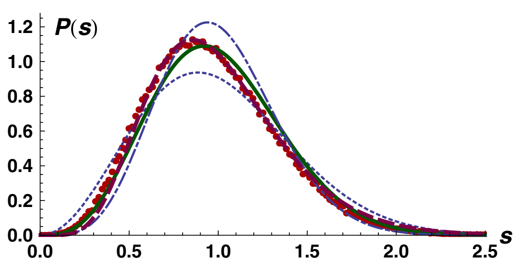

FIG. 1 (color online). Plots of LHE's numerical data [red dots] for the CZD ["$g(\alpha)$"] for $i = 1$ (their Fig. 1) and $P_n(s)$, $n = 2$ [dotted, blue line], 3 [solid, green line], and 4 [dash-dotted, blue line], along with $G_7(s) \propto s^6 e^{-7s}$ [dashed, purple line].

In [1], we assumed that the nucleation probability $\propto n^{i+1}$, where n is the adatom density. We then wrote $n \propto \bar{n}A/\langle A\rangle \equiv \bar{n}s$. Thus, the nucleation rate NR $\propto \bar{n}^{i+1}s^{i+1}$. But NR is also $\propto \bar{n}^{i+1}P(s)$. Thus, $P(s) \propto s^{i+1}$. SSA's and LHE's simulations imply that this argument is insufficient. We go beyond MF for small adatom coverage, thereby showing that larger exponents of s can arise.

In $2d$, the adatom density $n(r) \propto R^2 - r^2$, with $R_i < r < R$, where R and R_i are the radii of the CZ and island, respectively. Then, we find the total NR by integrating between these two radii, but $R_i \to 0$ for point islands, as well as for compact islands at small coverage; hence,

$$\int_{R_i\to 0}^{R} dr r[n(r)]^{i+1} \propto R^{2i+4} \propto A^{i+2} \Rightarrow P(s) \propto s^{i+2},$$

consistent with $\beta \approx 3$ (2) for $i = 1$ (0) in $2d$ [8].

The main points are that $P_\beta(s)$ accounts well for CZD, with physical information in β. The addend to i turns out to be larger than the MF prediction of 1, closer to 2, in this fascinating problem. In many experimental instances, the question is whether β changes, e.g., when impurities are added to the system [9].

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